How to Write an [M] Special Topic

M.Sc. in Mathematical Modelling & Scientific Computing,
Additional Skills

19th November 2025

Key Points

- go beyond the lectures, either by looking at more advanced mathematical models or investigating given models in more detail;
- additional details should be provided over and above what is given in the source materials. For example, extra steps in the calculations could be given, alternative methods could be used, or there could be an alternative derivation of a model along with a critical discussion of the modelling ideas;
- demonstrate that a range of sources has been consulted (and these should be referenced appropriately). A suitable critical literature review would be an appropriate form for a special topic report;
- summarise work at end and cite sources throughout.

Example: Further Mathematical Biology

One of the topics in the Further Mathematical Biology course concerns the formation of patterns via diffusion-driven instability.

One idea for a project is to investigate these models further, looking at the conditions required to generate a pattern and providing some numerical examples.

Title Page

The title page should consist of the title of the special topic, the name of the lecture course and your candidate number (NOT your name). You may also add a university crest. If this is all that is on the title page it does not contribute to the page count. If you add a table of contents or abstract then it does contribute to the page count. (Neither a table of contents nor an abstract is necessary.)

Title Page

Title of Special Topic

Lecture Course

Candidate Number: xxxxxx

Introduction

Give an introduction to the problem — what are you aiming to model, maybe a very brief history of the model (these ideas stem from Turing) and any relevant biology.

Derivation of the Model

In general for modelling special topics, the model will already exist so it is likely that your derivation will follow a book or paper — be sure to reference this. Then give a derivation of the model in your own words, making sure you introduce all notation as you use it and fill in any gaps in the reference(s) you are following.

If the derivation is long and there are several equations, it can be helpful to summarise them at the end of this section. Make sure you also give any appropriate initial and boundary conditions so that you have a complete model.

If the emphasis of your project will be more on the analysis and solution of the model, you do not need to derive it in great detail. You can just state the model and explain what different terms represent.

Reaction-Diffusion System

The problem to be studied is

$$\begin{aligned}\frac{\partial u}{\partial t} &= D_u \nabla^2 u + f(u,v) \\ \frac{\partial v}{\partial t} &= D_v \nabla^2 v + g(u,v)\end{aligned}$$

where the functions $f(u,v)$ and $g(u,v)$ need to be defined, along with initial and boundary conditions and the domain of interest.

Theory can be developed for a general domain and for general f and g, but then looking at particular kinetics for some simple domains and comparing analysis and numerical results makes a nice special topic.

Non-dimensionalisation

Non-dimensionalise the equations next, or give a reason why you don't! Non-dimensionalisation may help to identify different time-scales in a problem or help to see that some terms are very small and can perhaps be neglected.

Non-dimensional Schnakenberg Model

The dimensionless form of the Schnakenberg model is

$$\begin{aligned}\frac{\partial u}{\partial t} &= \gamma f(u,v) + \nabla^2 u,\\ \frac{\partial v}{\partial t} &= \gamma g(u,v) + d\nabla^2 v,\end{aligned}$$

where

$$\begin{aligned}f(u,v) &= a - u + u^2 v,\\ g(u,v) &= b - u^2 v,\end{aligned}$$

and a, b, and γ are positive constants and homogeneous Neumann boundary conditions are imposed on the boundary of the domain.

Can then find conditions on the parameters under which patterns will form.

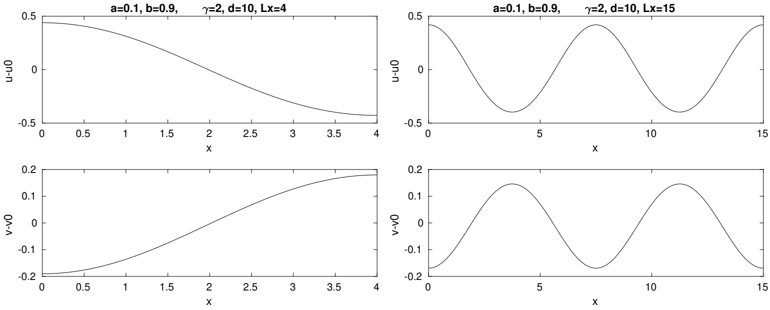
1D Example

Now pick some parameters. In 1D, could choose $a = 0.1$, $b = 0.9$, $d = 10$, $\gamma = 2$ and solve on the interval $[0, L_x]$. Then we have instability of those modes for which

$$0.4 \;\leq\; \frac{n^2\pi^2}{L_x^2} \;\leq\; 1.$$

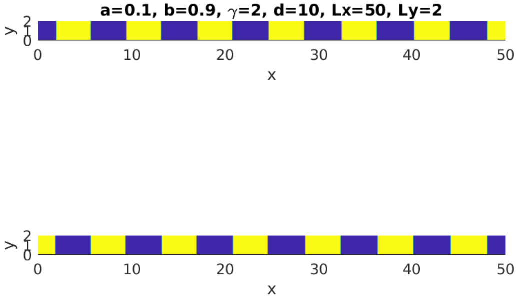
Choose L_x to get particular modes being unstable. Give enough details of the numerical simulations that they are reproducible, but remember the numerics is not the focus of the project.

1D Example

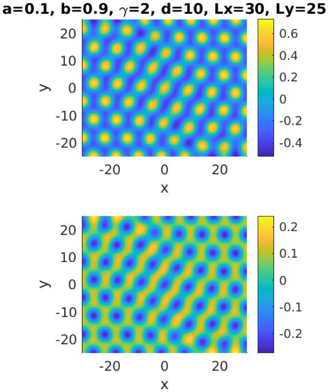

a=0.1, b=0.9, γ=2, d=10, Lx=4
a=0.1, b=0.9, γ=2, d=10, Lx=15
u-u0
v-v0
x
0.5
0
-0.5
0.2
0.1
-0.1
-0.2
1
1.5
2
2.5
3
3.5
4
5
10
15

2D Example

Now work on 2D rectangles. The types of pattern you get will vary according to how long and thin the rectangle is.



2D Example



How do the types of patterns you see relate the spots and stripes on animal coats?

(In [M] special topics you should always be trying to relate the mathematical results to the underlying problem.)

Conclusion

Make sure you write a summary of the project at the end, perhaps giving a critical discussion of the model(s) presented and suggestions for further investigation.

AI Declaration

All special topics must contain an AI declaration. This is *not* part of the page count. Even if you do not use AI, you need to provide a declaration saying so. The beginning of the special topic is a sensible place to put this so the assessor reads it before your project.

More details on how you can use AI are available in the departmental policy for students on the use of generative AI in summatively assessed work — see https://www.maths.ox.ac.uk/members/students/departmental-and-university-regulations.