String Theory 1

Lecture #1

Welcome to String Theory 1 !

→ xenia.delaossa@maths.ox.ac.uk

→ reading: see webpage of the course (good texts)

→ lecture notes: Ling Lin's from HT2023
(notes differences in conventions)

- My own notes will be uploaded to Moodle
- Sometimes I will use slides ← uploaded ahead of time (so you can annotate them)
- ∃ Index (table of contents) in Moodle

→ classes 3×2hour classes for each group (2 groups)

→ help cleaning the board @ the end of each lecture

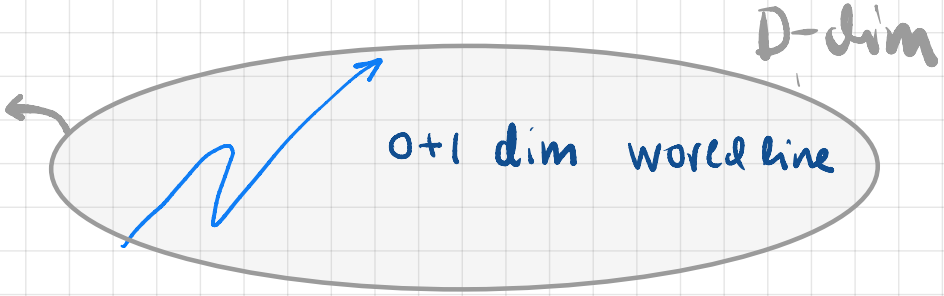
Chapter 0 Introduction as motivation

(Let's begin with a brief introduction as a way of motivation)

The starting point of string theory is that it is a theory of fundamental quantum mechanical strings

QFT: fundamental particles ↔ point-like objects •

particles trace trajectories in spacetime and we study the QFT on the WL (actions describing the physics of these particles)



String Theory: fundamental objects ↔ strings

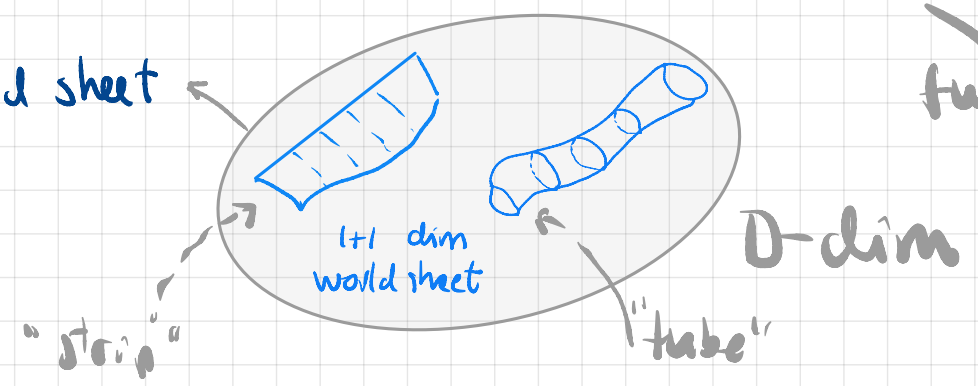
two topologies ~ or ○

open interval — closed loops

two "sectors"

string sweeps out a 1+1 dim world sheet in space time and we study QFTs on the WS of the string

↕ 2 dim 1+1



modern physical theories of nature are { Standard model, GR }

Before string theory (before 1985)

- **Standard model**: QFT of EM + weak forces + strong forces
  gauge theory with $U(1) \times SU(2) \times SU(3)$ symmetries
  - describes elementary particles and their fundamental interactions
  - ignores gravity: quantum mechanics + special relativity $M = \mathbb{R}^4$

standard model works at energy scales of $\sim 10^{-13}$ cm

gravity only becomes significant at much smaller scales $\sim 10^{-33}$ cm

A

each particle modeled by a point

is associated to a field

$\psi(x)$

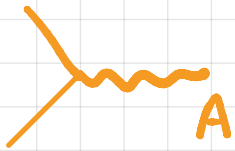
field

intrinsic properties (mass, e.m. charge, ...)

B

interactions between particles also described in terms of a particle $A^\mu(x)$ $(\mu = 0,1,2,3)$

spin 1 particle



math: interaction is localised in space time

↳ The standard model has been extremely successful! in its predictive power and is far consistent with experimental observations

↳ QFT is very rich in mathematical content (involves YM theories, Lie groups, ...)

Given its success mathematicians are trying to create a rigorous mathematical framework for QFT (axiomatic QFT → functional analysis, operator algebras, ...)

- **General relativity**: Einstein's theory of gravity
  - ↳ gravitational forces come from the curvature of spacetime caused by the presence of energy and matter
    - ↳ GR describes the phenomena associated to the large scale of the universe

10^{-13} cm

Mathematically intimately related to differential geometry where the gravitational field is the metric on spacetime & the theory is ruled by the principle of general covariance (physical theories independent of choice of coordinate systems)

Together (SM + GR): extremely successful theories of particle physics & gravity (consistent with experimental observations)

However there are many problems: (not complete list)

- SM leaves many questions unanswered:
  - The Lagrangian describing this model has too many (dimensionful) arbitrary parameters (~20, coupling constants, mass of particles) and there is no explanation for the values they take.

need to be added by hand

- Naturalness problem: no explanation for the disparate scales between SM & gravity
  - nuclear forces $\sim 10^{-13}$ cm
  - gravitational forces $\sim 1.6 \times 10^{-33}$ cm

$$L_P = \left(\frac{\hbar G_N}{c^3}\right)^{1/2} \qquad M_P = \left(\frac{\hbar c}{G_N}\right)^{1/2} = 1.1 \times 10^{19} \frac{\text{GeV}}{c^2}$$

- QFT (perturbative formulation) is incomplete:
  plagued by UV loop divergences when computing perturbative scattering amplitudes:
  - ↳ puts in doubt the validity of QFT at high energies

  however
  - ↳ we resort to regularisation and renormalisation (for renormalisable theories)

The standard model is a renormalisable QFT

- a quantum theory of gravity seems inconsistent (UV divergences!)

In fact the perturbative quantum field theory of gravity is **NOT** renormalisable

ST arose as an attempt to find a consistent quantum theory of gravity ~1984

Note however that S.T. was developed in the 60's to try to understand the behaviour of hadrons, in particular the large proliferation of hadronic resonances with higher & higher spin. This was abandoned in favor of QCD (new part of the S.M.). Also S.T predicted a massless spin 2 particle that wasn't observed in the hadronic spectrum

This is a first (introductory) course on string theory and we aim to discuss a few of its key features

- consistently incorporates gravity (GR) with quantum mechanics so maybe string theory is a theory of quantum gravity

In fact, (all) string theories contain in their massless spectrum at least one spin 2 boson that can interpreted as the graviton (ie the particle that mediates gravitational interactions)

↳ the closed string sector

- It incorporates other interacting & phenomenologically relevant ingredients from QFT & particle physics
  - non-Abelian gauge symmetries with chiral matter from the open string sector (except Het)

Note that a theory of open strings necessarily contains closed strings since open strings can close up

∴ quantum-gravity & YM theories (are in this sense "unified"
  ↳ ST contains all known forces of nature

- spacetime supersymmetry (ST 2 !)

- In some sense we get a unique theory (this is very subtle; relates to existence of dualities)

  There are no free parameters

  powerful! allows to discuss strong coupling in terms weak coupling

- Extra dimensions
  - $D = 26$ bosonic string
  - $D = 10$ superstrings

- extended objects ! (d-branes for example)

  (assumption that fundamental objects are strings
  ⇒ existence of extended objects in $d \neq 0$ dims)

  Non perturbative objects

But many unresolved issues

While it might be controversial to claim that ST is the fundamental theory of nature we still have many benefits from learning it:

- powerful tools to study strongly coupled field theories eg holography

  holographic principle ⟺ info re spectrum & dynamics of theory inside a volume V encoded in the degrees of freedom on ∂V

  AdS/CFT correspondence ⟺ weak-strong coupling duality

  ↳ further curves?

- ST has inspired and predicted new mathematics

  eg Mirror symmetry first discovered in ST

  very surprising mathematically

  ⋮ new branch of mathematics

  duality

This course: we will develop

- basic string theory
  - ↳ missing some of the features mentioned above and suffers from serious defects & inconsistencies eg contains tachyons
  - ↳ however illustrates key ideas & techniques in a relatively clean way.

If you can continue to

ST II: learn superstring theory

This is a more exciting theory.

Contents

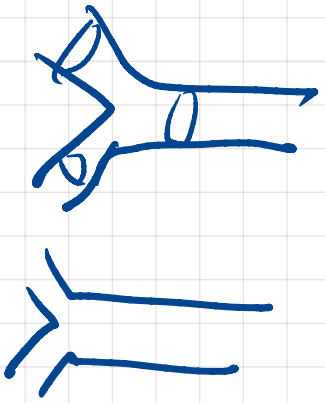
① Classical relativistic strings moving in ST

② Quantize strings
- spectrum ⊃ graviton
- space time dimensions: $D = 26$ as a consistency constraint of the quantum theory

2 dim theory on WS
↝ 2 dim CFT
↷ TT curve

③ Scattering of strings ↝ interactions

gets replaced by



All this on flat space time

④ Strings in background fields

↳ so far only WS 2dim theory

↳ how a $D = 26$ dim quantum gravity theory emerges

⑤ Compactifications & T-duality $D \to d$

↳ obtain a $d < 26$ dim theory

reduce 26 dim theory to an <u>effective</u> d dim theory

eg $M^{1,25}$ thru $M^{1,24} \times S^1$

branes appear

T-duality (most basic example of a duality)

Next

Chapter 1 Classical relativistic string

↳ study relativistic classical string propagating in a fixed spacetime M

⇝ 1.1 Classical relativistic point particle ↶ in a way that is generalisable to strings

1.2 Classical relativistic string: action principle

1.3 --