String Theory 1

Lecture #6

Chapter 2 Old covariant quantisation

There are several approaches (and the relation between them is not trivial)

1 Covariant BRST quantisation

modern path integral quantisation

$$Z = \int \frac{[\mathcal{D}X^\mu][\mathcal{D}\gamma]}{\mathrm{Vol}(\mathrm{Diff}\times\mathrm{Weyl})} e^{\frac{i}{\hbar} S_p[X^\mu,\gamma]}$$

This is the best quantum treatment of gauge theories.

(uses Fadeev-Popov-deWitt gauge fixing
& identifies BRST symmetries & currents
cancelation of Weyl anomaly requires D=26)

$\leadsto$ The right thing to do but requires more experience (AQFT) and takes longer.

2 Light-cone quantisation

Fix <u>all</u> gauge symmetry in the classical theory
(so Virasoro constraints are implemented classically)
But then the classical theory is not Poincaré invariant.

Hard work to see that anomaly cancels

see ST2

(subtleties eg $\sum_{n=1}^{\infty} n = -\frac{1}{12} = \zeta(-1)$)

3 Old covariant quantisation *

Start with the classical system in the conformal gauge and then quantise (promoting X^μ & Π^μ to operators, ...)

- One imposes the constraints $T_{\pm\pm} = 0$ on the quantum Hilbert space.

⇝ manifestly covariant

→ one needs $D = 26$ to cancel anomaly in Virasoro algebra

2.1 Introduction

Classical theory

$$S_p = -\frac{T}{2}\int_\Sigma d\tau\, d\sigma \left(-\partial_\tau X \cdot \partial_\tau X + \partial_\sigma X \cdot \partial_\sigma X\right)$$

in the conformal unit gauge $\gamma_{ab} = \eta_{ab}$.

This is supplemented by the constraints

$$T_{++} = 0 \quad \& \quad T_{--} = 0$$

impose after quantisation

The OCQ approach consists on promoting

the canonical fields X^μ
& their conjugate momenta $\Pi^\mu = T\partial_\tau X^\mu$ $\rightsquigarrow$ operators

and

the Poisson brackets $\{\cdot,\cdot\}_{PB}$ $\rightsquigarrow$ commutators of operators $i[\cdot,\cdot]$

We get the canonical equal time commutation relations

$$[\Pi^\mu(\tau,\sigma), X^\nu(\tau,\sigma')] = -i\,\delta(\sigma-\sigma')\,\eta^{\mu\nu}$$

(with $[X^\mu(\sigma), X^\nu(\sigma')] = 0$, $[P^\mu(\sigma), P^\nu(\sigma')] = 0$)

The operators X^μ & Π^μ are hermitian

$$X^\mu = (X^\mu)^\dagger, \qquad \Pi^\mu = (\Pi^\mu)^\dagger$$

this replaces the reality conditions of the classical fields

(operators X^μ and Π^μ must have real e-values!)

$\{x^\mu, p^\mu, \alpha_n^\mu, (\tilde{\alpha}_n^\mu)\} \rightsquigarrow$ are now operators

The commutation relations for the oscillator modes follow immediately from this:

$$[\hat{p}^\mu, \hat{x}^\nu] = -i\eta^{\mu\nu} \qquad \hat{p}^\mu, \hat{x}^\nu \text{ are Hermitian}$$

(Heisenberg algebra)

$$[\alpha^\mu_m, \alpha^\nu_n] = m\,\delta_{m+n,0}\,\eta^{\mu\nu} \qquad (\alpha^\mu_n)^\dagger = \alpha^\mu_{-n}$$

$$\left([\tilde{\alpha}^\mu_m, \tilde{\alpha}^\nu_n] = m\,\delta_{m+n,0}\,\eta^{\mu\nu} \qquad (\tilde{\alpha}^\mu_n)^\dagger = \tilde{\alpha}^\mu_{-n} \right)$$

This forms an infinite set of harmonic oscillators ($\alpha_m \to \frac{1}{\sqrt{m}}\alpha_m$) together with the Heisenberg pair $[x^\mu, p^\mu]$

Now we construct the Hilbert space in the usual way

2.2 Hilbert space (without constraints)

Identify oscillators (just as for the harmonic osc.)

$$\begin{cases} \alpha^\mu_{-n} \ (\tilde{\alpha}^\mu_{-n}), & n>0 \quad \text{raising} \\ \alpha^\mu_{n} \ (\tilde{\alpha}^\mu_{n}), & n>0 \quad \text{lowering} \end{cases}$$

Define the oscillator vacuum state $|0\rangle_{vac}$

$$\alpha^\mu_m |0\rangle_{vac} = \tilde{\alpha}^\mu_m |0\rangle_{vac} = 0 \quad \forall\, m>0$$

ie the state which is annihilated by all $\alpha^\mu_m (\tilde{\alpha}^\mu_m)$ $\forall\, m>0$

On top of $|0\rangle_{vac}$, we build the oscillator Fock spaces

ie states constructed by applying creation operators α^{μ}_{-n} $(\tilde{\alpha}^{\mu}_{-n})$, $n \geq 1$

$$\mathcal{H}^{Fock}_{open} = \mathrm{Span}\left\{ \prod_{i=1}^{k} \alpha^{\mu_i}_{-n_i} |0\rangle_{vac} \right\}_{n_i \geq 1}$$

$$\mathcal{H}^{Fock}_{closed} = \mathrm{Span}\left\{ \prod_{i=1}^{k} \alpha^{\mu_i}_{-n_i} \prod_{j=1}^{l} \tilde{\alpha}^{\mu_j}_{-n_j} |0\rangle_{vac} \right\}_{n_{i,j} \geq 1} = \underset{\mathcal{H}^{Fock}_{open}}{\underset{\cong}{\mathcal{H}^{Fock}_{R}}} \otimes \underset{\mathcal{H}^{Fock}_{open}}{\underset{\cong}{\tilde{\mathcal{H}}^{Fock}_{L}}}$$

It is useful to introduce oscillator number operators

$$N \equiv \sum_{k>0} \alpha_{-k} \cdot \alpha_k \qquad \tilde{N} \equiv \sum_{k>0} \tilde{\alpha}_{-k} \cdot \tilde{\alpha}_k$$

These statisfy:

$$[N, \alpha_n^\mu] = -n\, \alpha_n^\mu \qquad [N, \alpha_{-n}^\mu] = [N, (\alpha_n^\mu)^\dagger] = n\, \alpha_{-n}^\mu$$

(and similar for $[\tilde{N}, \tilde{\alpha}_n^\mu] = \cdots$)

N & $\tilde{N}$ are "counting" operators

$$N\left(\prod_{i=1}^{k} \alpha_{-n_i}^{\mu_i} |0\rangle_{vac}\right) = \left(\sum_{i=1}^{k} n_i\right)\left(\prod_{i=1}^{k} \alpha_{-n_i}^{k} |0\rangle_{vac}\right)$$

$$\tilde{N}\left(\prod_{i=1}^{k} \tilde{\alpha}_{-n_i}^{\mu_i} |0\rangle_{vac}\right) = \left(\sum_{i=1}^{k} n_i\right)\left(\prod_{i=1}^{k} \tilde{\alpha}_{-n_i}^{k} |0\rangle_{vac}\right)$$

Help organise oscillator states into "levels" (N & $\tilde{N}$ eigenstates)

Note that $N \geq 0$ (no states of negative level).

For open strings (or only R-movers closed string)

$N=0 \qquad |0\rangle_{vac}$

$N=1 \qquad \alpha^{\mu}_{-1}|0\rangle_{vac}$

$N=2 \qquad \alpha^{\mu}_{-2}|0\rangle_{vac}\,,\quad \alpha^{\mu_1}_{-1}\alpha^{\mu_2}_{-1}|0\rangle_{vac}$

$N=3 \qquad \alpha^{\mu}_{-3}|0\rangle_{vac}\,,\quad \alpha^{\mu_1}_{-2}\,\alpha^{\mu_2}_{-1}|0\rangle_{vac}\,,\quad \alpha^{\mu_1}_{-1}\alpha^{\mu_2}_{-1}\alpha^{\mu_3}_{-1}|0\rangle_{vac}$

$\vdots$

Hence

$$\mathcal{H}^{Fock}_{open} = \mathrm{span}\Big\{ \underbrace{\prod_{i=1}^{k} \alpha^{\mu_i}_{-n_i}|0\rangle_{vac}}_{\text{eigenstates of } N} \Big\}_{n_i\geq 1} = \bigoplus_{N=1}^{\infty} \mathcal{H}^{Fock}[N]$$

We are not done: we also have the zero modes $\{x^\mu, p^\nu\}$ (which commute with the oscillator modes).

We define the ground state (in momentum space)

$$|K, 0\rangle$$

with the property that it is an eigenvector of the momentum operator

$$\hat{p}^\mu |K, 0\rangle = K^\mu |K, 0\rangle \qquad K^\mu \in \mathbb{R}^{1, D-1}$$

& it is normalised such that $\langle K' | K\rangle = \delta^{(D)}(K - K')$

Then $\mathcal{H}_{\text{zero-mode}} \simeq L^2(\mathbb{R}^{1, D-1})$

$|K, 0\rangle$ in momentum space $\longleftrightarrow$ $\psi(x) \otimes |0\rangle_{\text{vac}}$

wave funct in spacetime

The Hilbert space is then

$$\mathcal{H}_{\text{open}} = L^2(\mathbb{R}^{1,D-1}) \otimes \mathcal{H}^{\text{Fock}}_{\text{open}} = \text{Span}\left\{ \prod_{i=1}^{K} \alpha^{\mu_i}_{-n_i} |k,0\rangle \right\}_{n_i \geq 1}$$

$$\mathcal{H}_{\text{closed}} = L^2(\mathbb{R}^{1,D-1}) \otimes \mathcal{H}^{\text{Fock}}_{L} \otimes \mathcal{H}^{\text{Fock}}_{R}$$

$$= \text{Span}\left\{ \prod_{i=1}^{h} \prod_{j=1}^{l} \alpha^{\mu_i}_{-n_i} \tilde{\alpha}^{\nu_i}_{-m_j} |k,0\rangle \right\}_{n_i, m_j \geq 1}$$

States labelled by spacetime momentum k and tensor indices, so they fall into representations of space-time Poincaré group

Problem (or not?): Consider the state

$$|\varphi\rangle = \alpha^0_{-1}|0;K\rangle \qquad \text{(level 1 state)}$$

Then

$$\langle\varphi|\varphi'\rangle = \langle 0;K|\underbrace{\alpha^0_{+1}}_{(\alpha^0_{-1})^\dagger}\alpha^0_{-1}|0;K'\rangle = \eta^{00}\,\delta(K-K')$$

$$= -\,\delta(K-K')$$

Wrong sign! There are negative norm states

(Ghosts!)

($\exists$ of negative norm states $\Rightarrow$ negative probabilities ???)

However: we have not imposed the constraints yet!

2.3 Constraints, normal ordering & Virasoro algebra

Recall the Witt-generators

$$L_m = \frac{1}{2} \sum_{k=-\infty}^{\infty} \alpha_{m-k} \cdot \alpha_k \,, \qquad L_m^{\dagger} = L_{-m} \quad (m \neq 0)$$

$$\tilde{L}_m = \frac{1}{2} \sum_{k=-\infty}^{\infty} \tilde{\alpha}_{m-k} \cdot \tilde{\alpha}_k \,, \qquad \tilde{L}_m^{\dagger} = \tilde{L}_{-m} \quad (m \neq 0)$$

quadratic in the oscillator operators : need a prescription for the ordering of the operators

The operators α_{m-k} & α_k commute $\forall k$ unless $m=0$

in which case $[\alpha^{\mu}_{-k}, \alpha^{\nu}_{k}] = 2k\, \eta^{\mu\nu}$.

Then only "problematic" operator is L_0 (& $\tilde{L}_0$)

(ie L_0 & $\tilde{L}_0$ are not determined by the classical expression)

Classically $L_0 = \frac{1}{2}\alpha_0^2 + \frac{1}{2}\sum_{n=1}^{\infty}(\alpha_{-n}\cdot\alpha_n + \alpha_n\cdot\alpha_{-n})$, $\tilde{L}_0 = \cdots$

Depending on how we order the raising/lowering operators (for each n) the action of L_0 (& $\tilde{L}_0$) on states can differ by a c-number (recall α's commute except $[\alpha_n^\mu, \alpha_{-n}^\nu] = n\eta^{\mu\nu}$)

To account properly for the normal ordering we $\boxed{\text{define}}$ the quantum operator L_0 as

$$L_0 \equiv \frac{1}{2}\sum_{n\in\mathbb{Z}} \underbrace{:\alpha_{-n}\cdot\alpha_n:}_{\substack{\text{normal ordered product} \\ \text{(lowering operator to the right)}}} \equiv \frac{1}{2}\alpha_0^2 + \sum_{n=1}^{\infty}\alpha_{-n}\cdot\alpha_n = \frac{1}{2}\alpha_0^2 + N$$

$L_0 - \tilde{L}_0 \propto \partial_\sigma$

breaks diffeo inv.

where now the quantum constraint is

$$\boxed{(L_0 - a)|\psi\rangle = 0 \qquad ((\tilde{L}_0 - \tilde{a})|\psi\rangle = 0)}$$

$\forall$ physical states $|\psi\rangle$

The constants a (and $\tilde{a}$) cannot be specified yet, but set $a = \tilde{a}$.

Virasoro algebra We need to check the commutator algebra of the operators L_m ($\tilde{L}_m$). A direct computation gives

PS 2

$$\boxed{[L_m, L_n] = (m-n) L_{m+n} + \frac{D}{12}(m^3-m)\,\delta_{m+n,0}}$$

(similar for $[\tilde{L}_m, \tilde{L}_n]$ for closed strings)

This is the Virasoro algebra with central charge D

We now call L_m the Virasoro generators.

The Virasoro algebra is a central extension of the Witt algebra by $c \in \mathbb{C}$

This can be expressed in terms of an exact sequence

central extension of Witt

$$
\begin{array}{ccccccccc}
0 & \longrightarrow & \hbar & \longrightarrow & \mathrm{Vir} & \longrightarrow & \mathrm{Witt} & \longrightarrow & 0 \\
 & & \nearrow & & \Uparrow & & \Uparrow & & \\
 & & \hat{c} : [\hat{c}, \hat{c}] = 0 & & \hat{c} = 0 & & \mathrm{Vir}/\langle \hat{c} \rangle & & \\
 & & [\hat{c}, L_n] = 0 & & & & & &
\end{array}
$$

The central extension is related to an anomaly of the Weyl invariance (more later).

[The global $sl(2)$ algebra generated by $\{L_0, L_1, L_{-1}\}$ (or $\{\tilde{L}_0, \tilde{L}_1, \tilde{L}_{-1}\}$] is not "anomalous".]

$m(m^2-1) = 0$

2.4 Constraints and physical states

↳ Imposing the constraints in the quantum theory to identify the physical states $\mathcal{H}_{phys} \subset \mathcal{H}$

<u>Problem</u>: $\boxed{\text{if}}$ we define $|\psi\rangle \in \mathcal{H}_{phys}$ to be states s.t.

$$L_m |\psi\rangle = 0$$

one finds a <u>contradiction</u>.

Consider $\quad [L_m, L_n]|\psi\rangle$

(a) $\quad [L_m, L_n]|\psi\rangle = (L_m L_n - L_n L_m)|\psi\rangle = 0 \quad \forall m, n$

OTOH

(b) $\quad [L_m, L_n]|\psi\rangle = \left[\cancel{(m-n)L_{m+n}} + \frac{D}{12}(m^3-m)\,\delta_{m+n,0}\right]|\psi\rangle$

$\neq 0 \quad m=-n$

So for $\quad n=-m$

$$[L_m, L_{-m}]|\psi\rangle = \frac{D}{12}(m^3-m)\,|\psi\rangle$$

Imposing all constraints leads to a trivial Hilbert space when $D \neq 0$!

Instead we define physical states $\phi, \psi \in \mathcal{H}_{phys}$ by the constraints

matrix elements of L_m vanish

$$\boxed{\langle \psi | L_m | \phi \rangle = 0} \qquad \forall m \neq 0$$

Definition: a state $|\phi\rangle$ is physical if

- $L_m |\phi\rangle = 0 \qquad \forall\ m \geq 1$
- $(L_0 - a)|\phi\rangle = 0 \qquad$ for fixed $a \in \mathbb{R}$

- $\{L_m\}_{m\geq 1}$ form a closed subalgebra $\left([L_m, L_n] = (m-n) L_{m+n},\ m,n \geq 1 \right)$
- $\langle \psi | L_m | \phi \rangle = \langle \phi | \underbrace{L_{-m} | \psi \rangle}_{=0}{}^{*} = 0 \qquad \forall \quad m \leq 1$

so $\quad L_m |\phi\rangle = 0 \ \ \forall m \geq 0$ incorporates all constraints $m \neq 0$

For closed string:

require the same constraints for $\tilde{L}_m$ for a state to be physical

Important remark: the generators of D-dim spacetime Poincaré symmetries have no normal ordering ambiguities

In fact

$$[P^\mu, L_m] = [M^{\mu\nu}, L_m] = 0 \qquad \forall m$$

$\therefore$ P^μ & $M^{\mu\nu}$ preserve the physical state conditions. This means that states in the phys decompose into representations of $SO(1,D-1)$ (covariant quantisation)

In summary: so far we have

$$\mathcal{H}_{phys} = \left[\bigcap_{m=1}^{\infty} Ker(L_m) \right] \cap Ker(L_0 - a)$$

normal ordering

$L_m|\psi\rangle = 0$ $\forall m \geq 1$

$(L_0 - a)|\psi\rangle = 0$

$\forall |\phi\rangle, |\psi\rangle \in \mathcal{H}_{phys}$:

$\langle\phi| L_m |\psi\rangle = 0 \quad \forall m \neq 0$

Situation is even simpler: L_{+1} & L_{+2} generate every possible L_m $m>2$ so only need $L_1|\psi\rangle = 0$ & $L_2|\psi\rangle = 0$!

$([L_1, L_2] = -L_3, \; [L_1, L_3] = -2L_4, \text{ etc} \dots)$

$$\boxed{\mathcal{H}_{phys} = Ker(L_2) \cap Ker(L_1) \cap Ker(L_0 - a)}$$

2.5 Mass-shell and level matching conditions

The L_0 (& $\tilde{L}_0$) conditions

Closed string

$L_0 = \frac{1}{2}\alpha_0^2 + N$ $\qquad$ $\alpha_0^\mu = \tilde{\alpha}_0^\mu = \sqrt{\frac{\alpha'}{2}}\, p$

mass shell condition

$$(L_0 + \tilde{L}_0 - 2a)|\varphi, k\rangle = 0 \iff \left(\frac{\alpha'}{2}k^2 + N + \tilde{N} - 2a\right)|\varphi, k\rangle = 0$$

so $\iff$ $\boxed{\alpha' M^2 = 2(N + \tilde{N} - 2a)}$

level matching condition

$$(L_0 - \tilde{L}_0)|\varphi, k\rangle = 0 \iff (N - \tilde{N})|\varphi, k\rangle = 0 \quad \text{so} \quad \boxed{N = \tilde{N}}$$

Open string

mass shell condition

$$(L_0 - a)|\varphi, k\rangle = 0 \iff (\alpha' p^2 + N - a)|\varphi, k\rangle = 0 \quad \text{so} \quad \boxed{\alpha' M^2 = N - a}$$

$\alpha_0^\mu = \sqrt{2\alpha'}\, p^\mu$

level 0 ie ground state $|0, k\rangle$

As an exercise, you can show that

$([N, \alpha^\mu_n] = -n\,\alpha^\mu_n)$

$$[N, L_m] = -m\,L_m$$

So L_m shifts N-level by $-m$ (similar for $\tilde{N}$ & $\tilde{L}$)

$N(L_m|\psi\rangle) = (n-m)\,|\psi\rangle$ for $N|\psi\rangle = n|\psi\rangle$

$\Rightarrow$ at level zero we only need to impose the L_0 conditions.

The L_0-conditions at level $N=0$ are

closed strings $\alpha' M^2 = -4a$

open strings $\alpha' M^2 = -a$

states with velocity > c ! q-instability to ground state not the true vac; maybe non-pert corrections fix this?

$a<0$	massive	ground state
$a=0$	massless	ground state
$a>0$	tachyonic	ground state (!)

2.6 Level 1 states & dealing with ghosts

Recall: earlier we encountered a problem with negative norm states (ghosts) at level one of the open string (eg: $\alpha^0_{-1}|0;K\rangle$).

We want to see if this issue remains after applying the constraints.

Consider a general level one open string state

$$\zeta \cdot \alpha_{-1}|0;K\rangle = |\zeta;K\rangle \qquad \zeta \in \mathbb{R}^{1,D-1}$$

polarization vector (D-degrees of freedom)

& impose the physical state conditions for L_0 & L_{+1}.

The L_n conditions for $n \geq 2$ are satisfied automatically once the ones for L_0 & L_{+1} are imposed

$$L_m \, \zeta \cdot \alpha_{-1}|0;K\rangle = \zeta_\mu [L_m, \alpha^\mu_{-1}]|0;K\rangle = \zeta \cdot \alpha_{m-1}|0;K\rangle$$

$$= 0 \qquad m \geq 2$$

- mass-shell condition $\underline{-\alpha' K^2 = \alpha' M^2 = 1-a}$

- L_{+1} condition:

$$L_{+1}(\zeta\cdot\alpha_{-1})|0;K\rangle = \eta_{\mu\nu}\zeta^\mu \underbrace{[L_{+1},\alpha^\nu_{-1}]}|0;K\rangle$$
$$= \eta_{\mu\nu}\zeta^\mu \alpha^\nu_0 |0;K\rangle = \sqrt{2\alpha'}\,\zeta\cdot K\,|0;K\rangle$$

$[L_m, \alpha^\mu_n] = -n\,\alpha^\mu_{m+n}$

$$L_{+1}|\zeta;K\rangle = 0 \iff \underline{\zeta\cdot K = 0}$$

hence ζ has D-1 independent components

- Norm of a general level 1 state

$$\langle\zeta;K|\zeta';K'\rangle = \langle 0;K|(\zeta\cdot\alpha_{+1})(\zeta'\cdot\alpha_{-1})|0;K'\rangle$$
$$= \zeta_\mu\zeta'_\nu\langle 0;K|[\alpha^\mu_{+1},\alpha^\nu_{-1}]|0;K'\rangle = \zeta\cdot\zeta'\,\delta(K-K')$$

For $\zeta=\zeta'$ require $\zeta^2\geq 0$ to avoid ghosts

↳ space like or null

Level 1 summary

$\alpha' K^2 = a-1$: so K is $\begin{cases} \text{spacelike if } a>1 \\ \text{lightlike if } a=1 \\ \text{timelike if } a<1 \end{cases}$

$\zeta \cdot K = 0$: transverse polarization

$\zeta^2 \geq 0$: polarization is Null or spacelike
to avoid ghosts

For $a > 1$, K is spacelike (L_0 condition)

Then the constraint $\xi \cdot K = 0$ (L_1 condition)

is satisfied by timelike polarisations ξ, $\xi^2 < 0$.

That is, we would get negative norm states (ghosts).

So for $a > 1$, the Virasoro constraints are not enough to eliminate them.

Then we reject $a > 1$ and require $a \leq 1$

Critical theory: Consider the case $a=1$ (threshold case)

$$a=1 \quad \Longrightarrow \quad K^2=0$$

$\Longrightarrow$ $|\zeta;K\rangle = \zeta\cdot\alpha_{-1}|0;K\rangle$ is a massless state

Additionally, there is a zero norm state

$\zeta=K$ $\quad |K;K\rangle = (K\cdot\alpha_{-1})|0;K\rangle$ w/ longitudinal polarisation

This state is orthogonal to $\boxed{\text{all}}$ physical states $|\zeta',K\rangle$

$$\langle K;K|\zeta,K'\rangle = (K\cdot\zeta)\,\delta(K-K') = 0$$ as $\zeta\cdot K'=0$ for physical states

Hence: the longitudinal polarisation decouples leaving $D-2$ physical polarisations like a photon!

However: for $a=1$ ground state is a tachyon

<u>Remark</u>: the decoupling of the longitudinal degree of freedom is due to the fact that it corresponds to a state that is "pure gauge":

$$L_{-1}|0;K\rangle = \sqrt{2\alpha'}\, K\cdot\alpha_{-1}|0;K\rangle = \sqrt{2\alpha'}\,|K;K\rangle$$

$$L_{-1}|0;K\rangle = \left(\frac{1}{2}\sum_{n=-\infty} \alpha_{-n-1}\cdot\alpha_n\right)|0,K\rangle = \frac{1}{2}\Big(\underset{k=0}{\alpha_{-1}\cdot\alpha_0} + \underset{k=-1}{\alpha_0\cdot\alpha_{-1}} + \sum_{n=2}^{\infty}\alpha_{n-1}\cdot\alpha_{-n}\Big)|0;K\rangle$$

$$= \alpha_{-1}\cdot\alpha_0|0;K\rangle = \sqrt{2\alpha'}\, K\cdot\alpha_{-1}|0;K\rangle$$

ie $|K;K\rangle$ is created by the action of L_{-1} which is a generator of a conformal transformation. In this sense we say that $|K;K\rangle$ is pure gauge state

↪ Virasoro constraints for $a=1$ precisely restrict the level 1 states to consistently describe space-time photons

$\underline{a < 1}$ $\quad K$ is timelike

$\xi \cdot K = 0 \Rightarrow \xi$ spacelike

(D-1 degrees of freedom)

$\xi^2 > 0$ +ve norm states

massive vector boson in D-dims.

Next: Ghosts & $D = 26$
$a = 1$