String Theory 1

Lecture # 9

Chapter 3 Interactions

3.1 Generalities

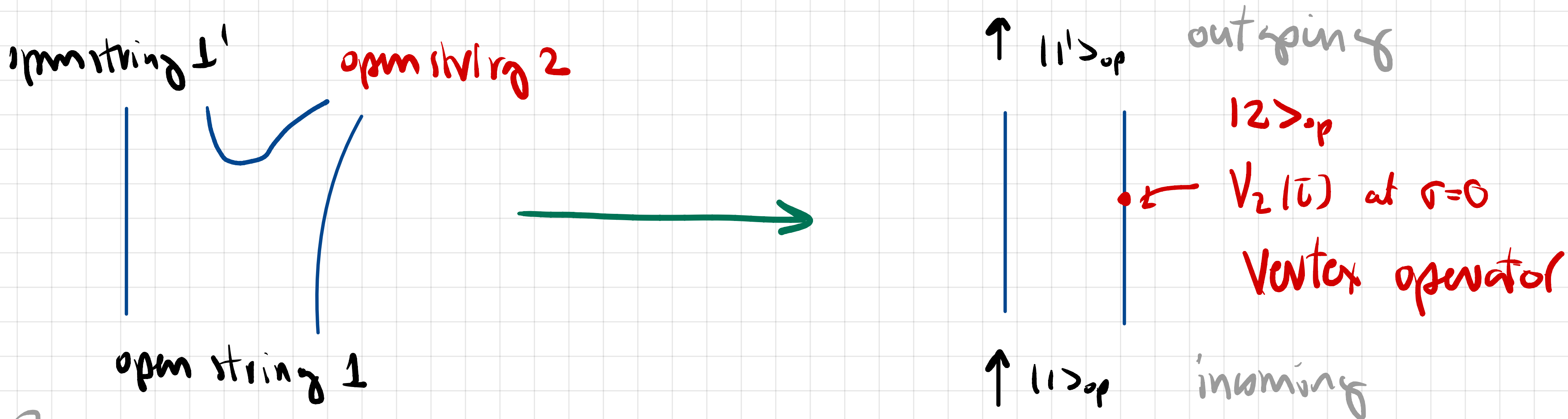
3.2 Vertex operators: introduction

3.3 Construction of vertex operators for the open string

3.4 State-Vertex correspondence (open string

3.5 Closed string

3.2 Introduction to vertex operators (summary)



not known how to do this

we instead describe emission/absorption of a quantum physical state from a fixed string world sheet by the action of a local operator or **vertex operator** V st

$$\mathcal{H}_{phys} \xrightarrow{V} \mathcal{H}_{phys} \qquad \mathcal{H}_{null} \xrightarrow{V} \mathcal{H}_{null}$$

Last lecture

$$\mathcal{H}_{\text{phys}} \xrightarrow{V} \mathcal{H}_{\text{phys}} \qquad \mathcal{H}_{\text{null}} \xrightarrow{V} \mathcal{H}_{\text{null}}$$

iff $$\boxed{[L_m, V(\tau)] = \partial_\tau \mathcal{O} \qquad \forall\, m \in \mathbb{Z}}$$

which is equivalent to the fact that $\int d\tau\, V(\tau)$ is invariant under conformal transformations $\tau \to \tilde{\tau}(\tau)$

Remark: $[L_m, V(\tau)] = \delta V(\tau)$ under conf transfs

ie integrated vertex op is invariant under $\tau \to \tilde{\tau}$

Next

For the open string a vertex operator is a primary operator of weight $h = 1$.

Recall from last lecture

Definition: an operator $A(\tau)$ is a primary operator of weight h if under the transformation $\tau \to \tilde{\tau}(\tau)$ it transforms as

$$\boxed{A(\tau) \longrightarrow \tilde{A}(\tilde{\tau}) = A(\tau)\left(\frac{d\tau}{d\tilde{\tau}}\right)^h}$$

For an operator with $h=1$

$$\int \tilde{A}(\tilde{\tau})\, d\tilde{\tau} = \int A(\tau) \frac{d\tau}{d\tilde{\tau}} d\tilde{\tau} = \int A(\tau)\, d\tau$$

the integrated operator is conformally invariant.

For infinitesimal transformations $\tau \longrightarrow \tilde{\tau} = \tau + \epsilon(\tau)$

$$\delta A(\tau) = -\partial_\tau(\epsilon A) - (h-1)\,\partial_\tau \epsilon\, A$$

This is a total derivative when $h=1$

But Virasoro generators generate (Work now on set $\ell = \begin{cases} 2\pi & \text{CS} \\ \pi & \text{OS} \end{cases}$)

$$\tau \longrightarrow \tilde{\tau} = \tau + \epsilon(\tau) \quad \text{with} \quad \epsilon(\tau) = -\epsilon\, e^{im\tau}$$

Then

Recall $\{L_m, \phi(\tau)\}_{PB} = \delta_{\epsilon_m}\phi(\tau)$ (PS 1)

$\hookrightarrow \delta_\epsilon \phi(\tau) = i\epsilon\, [L_m, \phi(\tau)]$

$$\delta\phi(\tau) = \epsilon\, e^{im\tau}\left(-i\partial_\tau \phi + h m \phi\right)$$

So the action of the Virasoro operators on primary ops is

$$\boxed{[L_m, \phi(\tau)] = e^{im\tau}(-i\partial_\tau + mh)\,\phi(\tau)}$$

Equivalently, this is the condition for ϕ to have conformal weight h.

For $h=1$: $[L_m, \phi(\tau)] = \partial_\tau(-i\, e^{im\tau}\phi(\tau))$ ✓ TD

The problem is to identify primaries of weight 1 which correspond to the emission/absorption of physical states in the string Hilbert space. Then we use this to compute string amplitudes!

3.3 Vertex operators for the open string

on the boundary of Σ

Consider the boundary scalar operator $X^\mu(\tau, 0)$:

$$X^\mu(\tau, 0) = x^\mu + 2\alpha' p^\mu \tau + i\sqrt{2\alpha'} \sum_{n \neq 0} \frac{\alpha_n^\mu}{n} e^{-in\tau}$$

WL of the end of the string

Conformal transformations:

$$[L_m, X^\mu(\tau, 0)] = -i e^{im\tau} \frac{d}{d\tau} (X^\mu(\tau, 0))$$

$$[L_m, \alpha_n^\mu] = -n \alpha_{m+n}^\mu$$

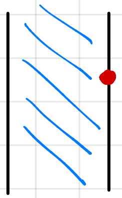
$\Rightarrow$ $\boxed{X^\mu(\tau) \text{ is an operator with } h=0 \text{ (indeed a WS scalar)}}$

but it is not a VO

Building a VO associated to a state of momentum K

$|\phi', q+k\rangle$ ⟷ out-state with momentum $q+k$

↑



$V_k(\tau)$ ← emission/absorption of a state with k

↑

$|\phi, q\rangle$ ← incoming state with q

so $V_K(\tau)\,|\phi; q\rangle = |\phi'; q+K\rangle$

(state with momentum k)

Thus, we require $[P^\mu, V_k(\tau)] = K^\mu V_k(\tau)$ action of the momentum tensor on an operator $V_k(\tau)$

Then the operator must depend on $e^{iK\cdot x(\tau)}$ ← CM coordinate

(QFT: know how to arrange this, this factor shifts momentum when acting on states)

A naive guess is: $V_k(\tau) \sim e^{iK\cdot X(\tau)}$ (simplest op built from X^μ)

As it stands, it is not well defined: it still needs normal ordering.

Consider the **normal ordered expression** (convention: annihilation ops on the right!)

$$\boxed{V_k(\tau) \equiv :e^{ik\cdot X(\tau)}: \equiv e^{\sqrt{2\alpha'}k\cdot\sum_{n=1}^{\infty}\frac{\alpha_{-m}}{m}e^{im\tau}}\, e^{ik\cdot x(\tau)}\, e^{-\sqrt{2\alpha'}k\sum_{n=1}^{\infty}\frac{\alpha_n}{n}e^{-in\tau}}}$$

$x^\mu(\tau) = x^\mu + 2\alpha'\hat{p}^\mu\tau$

$X^\mu(\tau,0) = \underbrace{x^\mu + 2\alpha' p^\mu\tau} + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{\alpha_n^\mu}{n}e^{-in\tau}$

$= x^\mu(\tau) + i\sqrt{2\alpha'}\sum_{n=1}^{\infty}\frac{1}{n}\left(\alpha_n^\mu e^{-in\tau} - \alpha_{-n}^\mu e^{in\tau}\right)$

There is still room for reordering eg

$$e^{ik\cdot x}\, e^{i2\alpha'\hat{p}\tau} \quad \text{vs} \quad e^{ik\cdot(x+2\alpha'\hat{p}\tau)} \qquad \text{etc}$$

we have chosen a particular ordering (the latter)

Baker-Campbell-Hausdorff

and also: $e^{ik\cdot\alpha_m}\, e^{ik\cdot\alpha_n} = e^{ik\cdot(\alpha_m+\alpha_n)}\, e^{-\frac{1}{2}k\cdot k\, m\,\delta_{m+n,0}}$

$k_\mu k_\nu[\alpha_m^\mu, \alpha_n^\nu] = m\,\delta_{m+n}\,\eta^{\mu\nu}$

so reordering is free for $k^2=0$ (useful soon!)

The normal ordering in the definition gives the right conformal transformation:

$$[L_m, :e^{ik\cdot X(\tau)}:] = e^{im\tau}\left(-i\partial_\tau + \alpha' m (k\cdot k)\right):e^{ik\cdot X(\tau)}:$$

PS 2

from reordering, gives $h = \alpha'(k\cdot k)$

This is a good vertex operator if $h=1$ ie $\alpha' k^2 = 1$ precisely the tachyon mass-shell condition ($\alpha' k^2 = 1$) !

This is the unique VO that describes emission/absorption of a tachyon.

Things get more interesting for $N=1$ states

Building vertex operators for level one states

For these states: $\underline{k^2=0}$

so we are looking for the photon emission/absorption vertex operator

$$W(\tau) \sim \underbrace{(\cdot \;- - -)}_{\text{some appropriate local operator}} \; \underbrace{: e^{ik\cdot X(\tau)} :}$$

$h = \alpha'(k\cdot k) = 0$

no normal ordering issues as $k^2=0$

eg $W(\tau)$ has $h=1$ after carefully considering the normal ordering

Consider $\partial_\tau X^\mu$:

$$[L_m, \partial_\tau X^\mu(\tau)] = \frac{\partial}{\partial\tau}[L_m, X^\mu(\tau)]$$

$$= \frac{\partial}{\partial\tau}\left(-i\left(\partial_\tau X^\mu(\tau)\right)e^{im\tau}\right) = e^{im\tau}\left(-i\,\partial_\tau + m\right)\partial_\tau X^\mu(\tau)$$

Then $\partial_\tau X^\mu(\tau)$ is primary of weight $h=1$ ✓

We could try for the vertex operator for emission/absorption of a photon with polarisation ς the operator

$$W_{\varsigma,k}(\tau) = \frac{1}{\sqrt{2\alpha'}}\left(\varsigma\cdot\partial_\tau X(\tau)\right):e^{ik\cdot X(\tau)}:$$

naively $h=1$ from $[L_m, AB]$

and deal with normal ordering issues.

Note however that AB does not necessarily have $h_{AB} = h_A + h_B$ ...

Naively the VO for the tachyon has weight 0 but NO $\Rightarrow$ it has $h=1$

$h_{AB} = h_A + h_B$ only for ops unambiguously defined (no sing. as $\tau\to\tau'$ in $A_\tau B_{\tau'}$

Another example: $:X^\mu(\tau)X^\nu(\tau):$ has no definite h.

$$\text{In } \overbrace{\xi \cdot \partial_\tau X(\tau)}^{\sqrt{2\alpha'} \sum_n \xi \cdot \alpha_n e^{-in\tau}} \text{ : each oscillator operator is contracted with } \xi$$

$$\text{In } e^{iK \cdot X(\tau)} \text{ : each oscillator operator is contracted with } K$$

Then, all potential ambiguities come from commutators of the form:

$$[\alpha_m \cdot \xi, \alpha_n \cdot K] = m\, \delta_{m+n,0}\, (\xi \cdot K).$$

Precisely when $\xi \cdot K = 0$ (physical polarization)

$$(\xi \cdot \partial_\tau X(\tau)) :e^{iK \cdot X(\tau)}: \text{ is well defined}$$

ie no normal ordering ambiguities!

We identify

$$\boxed{W_{s,k}(\tau) = \frac{1}{\sqrt{2\alpha'}}\,:(\varsigma\cdot\dot{X}(\tau))\,e^{ik\cdot X(\tau)}: , \quad k^2=0,\ \varsigma\cdot k=0 \quad (h=1)}$$

with the vertex operator for the emission/absorption of a photon.

Note that, for the longitudinal polarization,

$$(k\cdot\dot{X})\,e^{ik\cdot X} = -i\,\partial_\tau\left(e^{ik\cdot X}\right)$$

which vanishes after integrating over τ.

This means that the longitudinal mode decouples

It is not a coincidence that the rules for constructing vertex operators are closely parallel to the construction of physical states.

In summary: $|\phi\rangle_{\text{phys}} \longmapsto V_\phi(\tau)$ ← conformal primary operator of weight $h=1$

For example, recalling that $:e^{ik\cdot X}:$ has $h = \frac{1}{2}(k\cdot k)\alpha'$

level 0 : $|0;k\rangle \longmapsto\ :e^{ik\cdot X(\tau)}:$ $h=1$ if $\alpha' k^2 = 1$

tachyon

level 1 : $|\zeta;k\rangle \longmapsto\ :\zeta\cdot\dot{X}(\tau)\, e^{ik\cdot X(\tau)}:$ $h=1$ if $k^2=0$ & $\zeta\cdot k = 0$,

photon of polarisation ζ

transverse polarisation

(longitudinal mode decouples $k\cdot\dot{X}\, e^{ik\cdot X(\tau)} = -\partial_\tau\left(e^{ik\cdot X}\right)$)

level 2 : $\gamma_{\mu\nu}\,\alpha_{-1}^{\mu}\,\alpha_{-1}^{\nu}\,|0;k\rangle \longmapsto \underbrace{\gamma_{\mu\nu}\,\dot{X}^\mu \dot{X}^\nu}_{h=2} \underbrace{:e^{ik\cdot X}:}_{h=-1}$ $h=1$ if $k^2 = -2\alpha'$ & $k\cdot\gamma = 0$, γ traceless

massive spin 2 state (symmetric traceless tensor)

One can show for $\forall N \geq 0$ that for every physical state in $\mathcal{H}_{phys}$ one can construct a corresponding VO (by careful consideration of the normal ordering & demanding $h=1$)

The converse is also true: every VO corresponds to a physical state!

↳ Next: The state-operator correspondence (special feature of CFTs)

3.4 State-Vertex correspondence (open strings)

We want to make this correspondence more explicit:

$$|\phi\rangle = \text{the phys} \quad \xleftrightarrow{1-1} \quad V_\phi$$

primary operator of weight $h=1$ (ie Vertex operators)

We begin recalling that the Hamiltonian

$$H = L_0 = \alpha' p^2 + N$$

is a time evolution operator. So, if $V(\tau)$ is a vertex operator at the string end point $\sigma=0$, then we can write

$$V(\tau) = e^{i\tau L_0} V(0) e^{-i\tau L_0}$$

& $V(\tau)$ describes the emission/absorption of a physical state from the end of the open string ($\sigma=0$) at τ if $h=1$.

For the tachyon: $V_T(k,\tau) = :e^{ik\cdot X(\tau)}: = e^{i\tau L_0} \underbrace{:e^{ik\cdot X(0)}:}_{V_T(k,0)} e^{-i\tau L_0}$

The point now is that starting from this, one can recover the tachyon state $|k;0\rangle$.

Consider the action of this operator on "zero momentum" state $|0;0\rangle$:

$L_0 = \frac{1}{2}\alpha_0^2 + N$

$$V_T(k,\tau)|0;0\rangle = e^{i\tau L_0}\left(:e^{ik\cdot X(0)}: \underbrace{e^{-i\tau L_0}\right)|0;0\rangle}_{|0;0\rangle \text{ as } L_0|0;0\rangle = 0}$$

$ik\cdot X(0,0) = ik\cdot x - \sqrt{2\alpha'} \sum_{\substack{n\in\mathbb{Z} \\ n\neq 0}} \frac{1}{n} k\cdot\alpha_n^\mu$

$$V_T(k,\tau)|0;0\rangle = e^{i\tau L_0}\left(e^{\sqrt{2\alpha'}\sum_{n\geq 1}\frac{1}{n}k\cdot\alpha_{-n}} \underbrace{:e^{ik\cdot x(0)}: \underbrace{e^{-\sqrt{2\alpha'}\sum_{n\geq 1}\frac{1}{n}k\cdot\alpha_n}\right)|0,0\rangle}_{|0,0\rangle \text{ as } \alpha_n|0,0\rangle = 0 \ \forall n\geq 1}}_{|0;k\rangle}$$

So $V_T(K,\tau)|0;0\rangle = e^{i\tau L_0}\, e^{\sqrt{2\alpha'}\sum_{n\geq 1}\frac{1}{n}K\cdot\alpha_{-n}}\,|0;K\rangle$

Wick rotation: pass to imaginary or Euclidean time on the WS

Define $\quad z = e^{i\tau} = e^{t} \quad$ (where $\tau = -it$)

so t is Euclidean WS time

Then:

$L_0 = \alpha' p^2 + \hat{N}$

$$V_T(K,\tau)|0;0\rangle = z^{L_0}\, e^{\sqrt{2\alpha'}\sum_{n\geq 1}\frac{1}{n}K\cdot\alpha_{-n}}\,|0;K\rangle$$

$$= z^{1+\hat{N}}\Big(1+\underbrace{\sqrt{2\alpha'}\,K\cdot\alpha_{-1}}_{N=1}+\frac{1}{2}\sqrt{2\alpha'}\underbrace{\big(K\cdot\alpha_{-2}+\sqrt{2\alpha'}\,(K\cdot\alpha_{-1})^2\big)}_{N=2}+\cdots\Big)|0;K\rangle$$

$\alpha' k^2 = 1$

$$= z\Big(1+\sqrt{2\alpha'}\,z\,K\cdot\alpha_{-1}+2\,\frac{1}{2}\sqrt{2\alpha'}\,z^2\big(K\cdot\alpha_{-2}+\sqrt{2\alpha'}\,(K\cdot\alpha_{-1})^2\big)+\cdots\Big)|0;K\rangle$$

We can recover the tachyon state $|0;K\rangle$ from V_T in the limit:

$$\lim_{z\to 0}\frac{1}{z}V_T(K,\underbrace{-i\log z}_{\tau})|0;0\rangle = \lim_{t\to -\infty} e^{-t}\,V_T(K;it)\,|0;0\rangle = |0;K\rangle \quad \checkmark$$

past infinity

For the photon

$L_0 = \alpha' p^2 + N$

$V_S(K,\tau) = S\cdot\dot{X}\, e^{iK\cdot X}$

normal ordered when $\zeta\cdot K = 0$

$$V_S(K,\tau)|0;0\rangle = \frac{1}{\sqrt{2\alpha'}}\, e^{i\tau L_0}\, \underbrace{V_S(K,0)\, e^{-i\tau L_0}|0;0\rangle}$$

$$= \frac{1}{\sqrt{2\alpha'}}\, e^{i\tau(\alpha' p^2 + N)}\, \underbrace{\zeta\cdot\dot{X}(0)\, e^{iK\cdot X(0)}\, \underbrace{e^{-i\tau L_0}|0;0\rangle}_{|0;0\rangle}}_{|0;K\rangle}$$

as $k^2=0$

$$= \frac{1}{\sqrt{2\alpha'}}\, z^N\, (\zeta\cdot\dot{X}(\tau=0))|0;K\rangle = z^N \sum_{m>0} (\zeta\cdot\alpha_{-m})\, e^{\sum_{n>0}\frac{1}{n}K\cdot\alpha_{-n}}|0;K\rangle$$

$$= \Big(\underbrace{z(\zeta\cdot\alpha_{-1})}_{N=1} + \underbrace{z^2\big(\zeta\cdot\alpha_{-2} + (\zeta\cdot\alpha_{-1})(\zeta\cdot\alpha_{-1})\big)}_{N=2} + \cdots\Big)|0,K\rangle$$

We then recover the state $|\zeta;K\rangle$:

$$|\zeta;K\rangle = \lim_{z\to 0}\frac{1}{z} W_{\zeta,K}(-i\log z)|0;0\rangle$$

$$= \lim_{t\to-\infty} e^{-t}\, W_{\zeta,K}(it)|0;0\rangle$$

More generally,

$$|\psi\rangle \in \mathcal{H}_{\text{phys}} \xleftrightarrow{1-1} V_\psi(\tau) \quad \text{with}$$

$$|\psi\rangle = \lim_{z\to 0} \frac{1}{z} V_\psi(-i\log z)|0;0\rangle$$

$$= \lim_{t\to -\infty} e^{-t} V_\psi(it)|0;0\rangle$$

We can treat annalogously the "out going" states. For the tachyon, for example:

$$\langle 0;0|\, V_T(K, i\tau) = \langle 0;K|\, e^{\sqrt{2\alpha'}\sum_{n>0}\frac{1}{n}K\cdot\alpha_n}\, e^{-i\tau L_0} \qquad z^{-L_0} = z^{-1-N}$$

$$= \frac{1}{z}\langle 0;K|\left(1+\sqrt{2\alpha'}\, z^{-1}(\alpha_1\cdot K) + 2\,|\alpha'|\frac{1}{2}\, z^{-2}\,(\alpha_2\cdot K + (K\cdot\alpha_1)(K\cdot\alpha_1)) + \cdots\right)$$

$$\Rightarrow \langle 0;K| = \lim_{z\to\infty} z\,\langle 0;0|\, V_T(K, -i\ln z) = \lim_{t\to\infty} e^{t}\langle 0;0|\, V_T(K, -i\tau)$$

In summary:

incomming state $$|\psi\rangle = \lim_{z \to 0} \frac{1}{z} V_\psi(-i \log z) |0;0\rangle$$

(ie recover an incomming state $|\psi\rangle$ by acting with a VO on $|0;0\rangle$ & taking the Euclidean past infinite limit)

$$\langle \phi | = \lim_{z \to \infty} z \langle 0;0| V_\psi(-i \log z)$$

(ie recover an outgoing state $\langle \phi|$ by acting with a VO on $\langle 0;0|$ & taking the Euclidean future infinite limit)

Remark: In 2dim CFT we have we have more generally a state - operator correspondence where

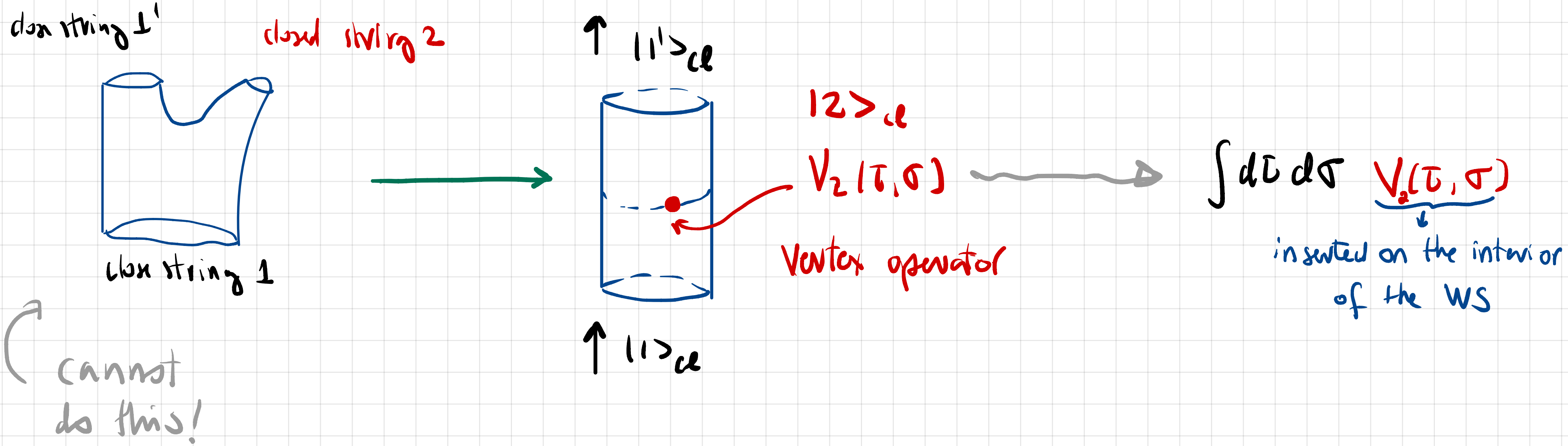
$$|\psi_A\rangle \xleftrightarrow{1-1} A(\tau)$$

irred reps of Vir-alg — primary operator of weight h_A

$$|\psi_A\rangle = \lim_{t \to -\infty} z^{h_A} A(it) |0,0\rangle$$

$$\langle \psi_A| = \langle 0,0| \lim_{t \to \infty} z^{-h_A} A(it)$$

3.5 Vertex operators for closed strings



we describe emission/absorption of a quantum physical state from a fixed string world sheet by the action of a local operator or vertex operator V st

$$\mathcal{H}_{phys} \xrightarrow{V} \mathcal{H}_{phys} \qquad \mathcal{H}_{null} \xrightarrow{V} \mathcal{H}_{null}$$

A primary operator of dimension $(h, \tilde{h})$ is an operator transforming under conformal transfs of the WS according to

$$\phi(\xi^+, \xi^-) \longrightarrow \tilde{\phi}(\tilde{\xi}^+, \tilde{\xi}^-) = \left(\frac{d\xi^+}{d\tilde{\xi}^+}\right)^h \left(\frac{d\xi^-}{d\tilde{\xi}^-}\right)^{\tilde{h}} \phi(\xi^+, \xi^-)$$

The corresponding infinitesimal transformations are

$$\delta\phi(\tau, \sigma) = -\partial_+(\tilde{\epsilon}\phi) - (\tilde{h}-1)(\partial_+\tilde{\epsilon})\phi - \partial_-(\epsilon\phi) - (h-1)(\partial_-\epsilon)\phi$$

This is a total derivative if $h = \tilde{h} = 1$

For $\epsilon_n^\pm = \frac{i}{2} e^{2im\xi^\pm}$ this gives the action of L_m:

$$[L_m, \phi(\xi^\pm)] = \frac{1}{2} e^{2im\xi^+} \left(-i\partial_+ + 2mh\right) \phi(\xi^\pm)$$

$$[\tilde{L}_m, \phi(\xi^\pm)] = \frac{1}{2} e^{2im\xi^-} \left(-i\partial_- + 2m\tilde{h}\right) \phi(\xi^\pm)$$

Vertex operators : $\underline{h = \tilde{h} = 1}$

The closed string operator $\quad :e^{ik\cdot X(\xi^{\pm})}:$

is primary with $h = \tilde{h} = \frac{\alpha'}{4} k\cdot k$ so

$$V_T(k;\xi^{\pm}) = :e^{ik\cdot X(\xi^{\pm})}:$$

VO $(h=\bar{h}=1)$ corresponding to the tachyon $\alpha' k^2 = 4$

For higher levels

$$V(k;\xi^{\pm}) = (\cdots) :e^{ik\cdot X(\xi^{\pm})}:$$

$L_0 = \frac{1}{2}\alpha_0^2 + N \qquad \alpha_0^{\mu} = \tilde{\alpha}_0^{\mu} = \sqrt{\frac{\alpha'}{2}}\, p$

$$\boxed{\alpha' M^2 = 2(N + \tilde{N} - 2a)}$$

As for the open string: with the appropriate normal ordering one can construct vertex operators $(h=\bar{h}=1)$ for each $|\varphi\rangle \in \mathcal{H}_{phys}$

For the converse: given a Vertex operator $(h = \tilde{h} = 1)$ one can recover a physical state.

If $V(\tau, \sigma)$ is a VO then we can write

translation in $\xi^{\pm}$ $<$ $L_0 + \tilde{L}_0 \sim \partial_\tau$, $L_0 - \tilde{L}_0 \sim \partial_\sigma$

$$V(\xi^{\pm}) = e^{2i\xi^{-} L_0 + 2i\xi^{+}\tilde{L}_0}\, V(0,0;k)\, e^{-2i\xi^{-} L_0 - 2i\xi^{+}\tilde{L}_0}$$

insertion of vertex at the origin $\xi^{\pm} = 0$

Wick rotation to Euclidean time $\tau = -it$:

light-cone coordinates: $\xi^{\pm} = -i(t \pm i\sigma)$

and let $z = e^{i\xi^{+}}$, $\bar{z} = e^{i\xi^{-}}$ $\quad \bar{\xi}^{+} = -\xi^{-}$

The state vertex correspondence is given by

$$|\psi\rangle = \lim_{t\to-\infty} e^{-4t}\, V_\psi[-it,\sigma]\, |0,\tilde{0};0\rangle$$

$$= \lim_{|z|\to 0} |z\bar{z}|^{-2}\, V_\psi\left(-\frac{i}{2}\log(z\bar{z}),\ -\frac{i}{2}\log\left(\frac{z}{\bar{z}}\right)\right) |0,\tilde{0};0\rangle$$

$$z = e^{i\xi^+}, \quad \bar{z} = e^{i\xi^-}$$

$$z\bar{z} = e^{2t} \qquad \frac{z}{\bar{z}} = e^{2i\sigma}$$

so $|\psi\rangle$ recovered by the insertion of VO

at euclidean past-infinity

$\hookrightarrow$ 3 point interactions (tree level)