String Theory 1

Lecture # 16

6 D-branes

Last lecture: we defined a Dp-brane as a (p+1)-dimensional subspace of target space where the ends of open strings can end

(We refer to this subspace as the D brane world volume)

We saw how D-branes appear from T-duality

strings with Neumann boundary conditions $\longleftrightarrow$ Dirichlet boundary conditions

Today: a number of observations about Dbranes (mostly without proofs $\rightsquigarrow$ just an idea of what these important objects are in the context of string theory; see eg Zwiebach)

In this lecture course we studied:

- quantised strings (open & closed) in $\mathbb{R}^{1,25}$

  a salient feature is that the massless sector includes

  a graviton (from the closed sector)

  gauge fields (from the open sector)

  Note that we discussed OS with Neumann bcs only

- We also discussed quantised strings in $\mathbb{R}^{1,24} \times S^1_R$

  ↳ new features eg

  CS • states have quantised momentum along S^1_R and a winding quantum number

  • T duality

  OS more complicated; T-duality leads to the notion of D-branes

Last lecture:

OS - T-duality $\longleftrightarrow$

open string with Neumann boundary conditions compactified on S^1_R

dual open string with Dirichlet boundary conditions compactified on $S^1_{\hat{R}}$, $\hat{R} = \alpha'/R$

D25 space-filling brane
↳ open string ends are free to move on spacetime

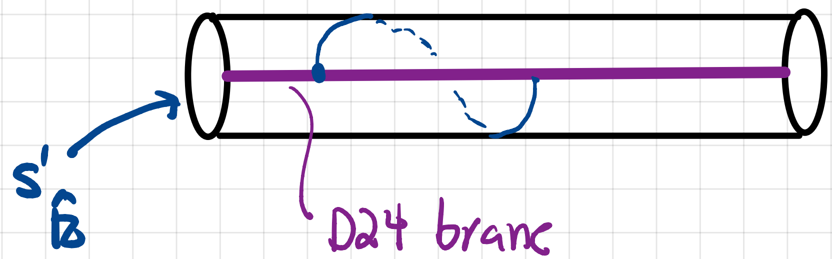
endpoints of the string live on a D24 brane

$p^{25} = \frac{m}{R}$ quantized

no translational symmetry along $S^1_{\hat{R}}$

no winding

string can wind around $S^1_{\hat{R}}$

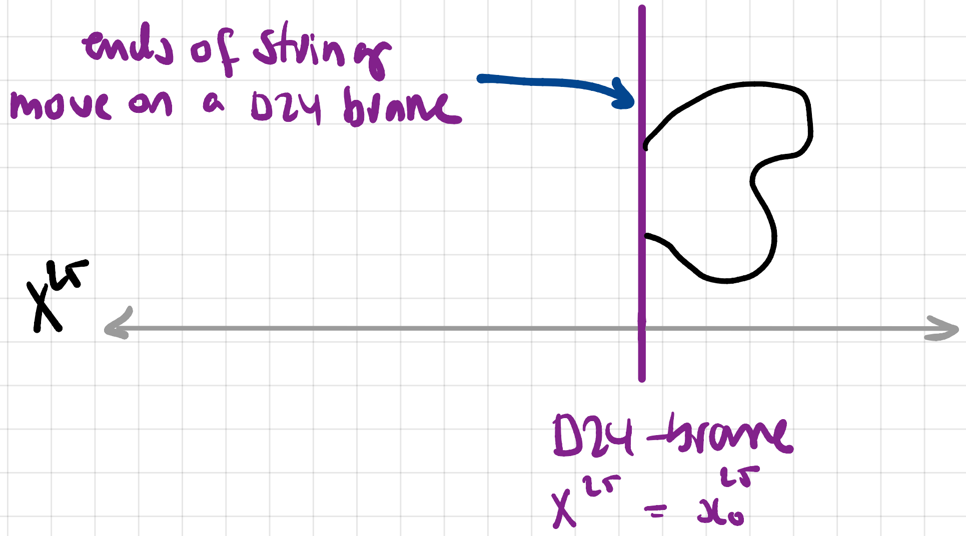


massless sector: (both sides)
25 dimensional U(1) gauge field

$\boxed{6.1}$ Open strings with Dirichlet b.cs in $\boxed{\text{flat}}$ $\mathbb{R}^{1,25}$

(no compactification)

Consider an open string on $\mathbb{R}^{1,25}$ with Dirichlet boundary conditions in $\boxed{\text{one}}$ direction (X^{25}) and Neumann boundary conditions in all other directions (X^i $i=0,\dots,24$).



no translational symmetry along X^{25}

space-time symmetry

$$SO(1,25) \longrightarrow SO(1,24)$$

[More generally, one can consider an open string with Dirichlet boundary conditions in $26-(p+1)$ directions and Neumann boundary conditions in $(p+1)$ directions. In this case string ends move on a Dp brane and $SO(1,25) \rightarrow SO(1,p) \times SO(25-p)$]

Mode expansion for $X^\mu(\tau,\sigma)$:

Neumann boundary conditions

$$X^i(\tau,\sigma) = x^i + 2\alpha'\tau p^i + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha_n^i \cos(n\sigma)\, e^{-in\tau} \qquad i=0,\ldots,24$$

Dirichlet boundary conditions

$$X^{25}(\tau,\sigma) = x_0^{25} + \frac{1}{\sqrt{2\alpha'}\,\pi}\left(x_1^{25}-x_0^{25}\right)\sigma + \sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha_n^{25}\, e^{-in\tau}\sin(n\sigma)$$

0

$$X^{25}(\tau,0) = X^{25}(\tau,\pi) = x_0^{25}$$

$\boxed{\text{no } \alpha_0^{25} \text{ mode}}$.

$\Rightarrow$ no momentum along X^{25}

(a term $p^a\tau$ $\Rightarrow$ endpoints would not stay at x_0^{25} when $\tau\neq 0$)

quantizing the string: mostly as before except

X_0^{25} remains a number

(X_0^{25} is not a parameter, it represents the location of a fixed D-brane)

Virasoro operators as before.

Mass-shell condition: $L_0 - 1 = (\alpha' p^2 + N) - 1$ all oscillators

becomes $\alpha' M_{25}^2 = -\alpha' |p|^2 = N-1$, $|p|^2 = p \cdot p$ inner product on $\mathbb{R}^{1,24}$

Ground level $(N=0)$: tachyon on the D-brane $\alpha' M_{25}^2 = -1$

Massless spectrum: level $N=1$

$$|\xi,\eta;k\rangle = (\xi\cdot\alpha_{-1} + \eta\,\alpha^{25}_{-1})\,\overbrace{|0;k\rangle}^{\text{ground state }(N=0)}$$

k: 25 dim momentum

ξ: $(1+24)$-dim polarization vector

η: spacetime scalar

Imposing $L_1|\xi,\eta;k\rangle = 0$ $\quad(\Rightarrow L_m|\phi\rangle = 0,\ m\geq 2)$

we find that $|\xi,\eta;k\rangle$ is physical if $\xi\cdot k = 0$

with η unconstrained.

$$\underline{L_1|\xi,\eta;k\rangle} = \left(\xi_i\left([L_1,\alpha^i_{-1}] + \alpha^i_{-1}L_1\right) + \eta\left([L_1,\alpha^{25}_{-1}] + \alpha^{25}_{-1}L_1\right)\right)|0;k\rangle$$

$$= \left(\xi\cdot\alpha_0 + \cancel{\eta\,\alpha^{25}_0} + (\xi\cdot\alpha_{-1} + \eta\,\alpha^{25}_{-1})L_1\right)|0;k\rangle$$

$$= \left(\xi\cdot k + \cancel{(\xi\cdot\alpha_{-1} + \eta\,\alpha^{25}_{-1})(\alpha_1\cdot\alpha_0)}\right)|0;k\rangle = \underline{(\xi\cdot k)|0;k\rangle}$$

null states at level one of the form $L_{-1}|0;K\rangle$:

$$L_{-1}|0;K\rangle = K\cdot\alpha_{-1}|0;K\rangle \quad \text{with} \quad K\cdot K=0$$

no osc.

Then we have the massless physical states

- 25-dimensional photon $\quad \xi\cdot\alpha_{-1}|0;K\rangle$ $\quad$ u(1) physical states
  $\xi\cdot K=0$

so the Dbrane has a u(1) field on its worldvolume (true for any Dp brane)

- scalar field $\quad \varphi = \eta\,\alpha_{-1}^{25}|0;K\rangle$

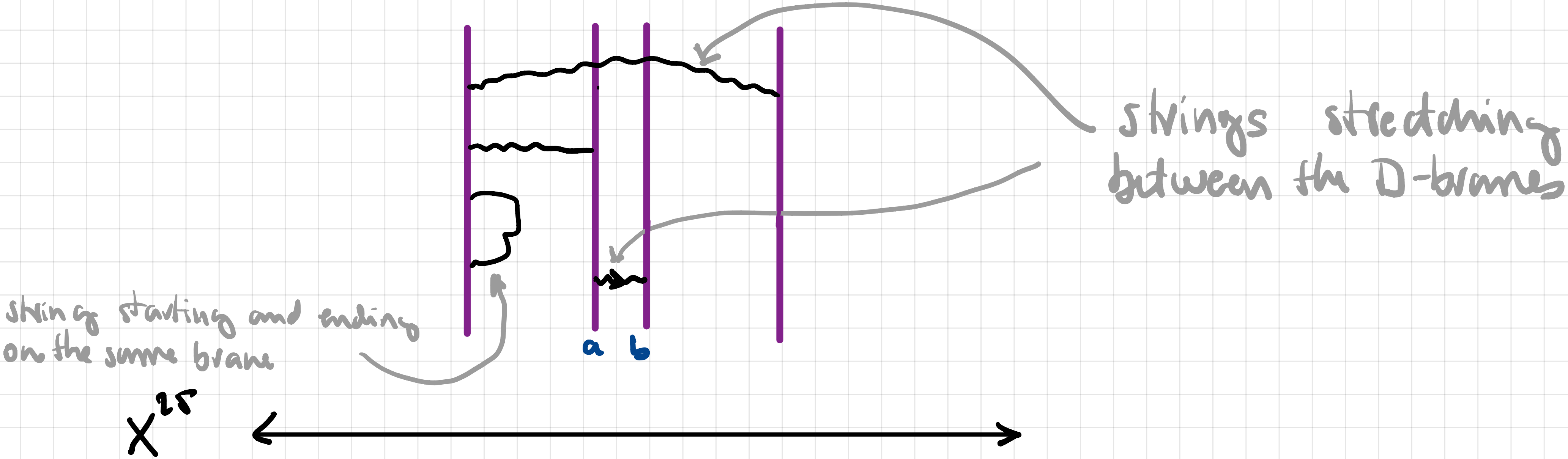
more generally a Dp brane has a massless scalar for each normal direction.

φ can be identified with fluctuations in the position of the D-brane along the transverse X^{25} direction (no proof here!)

see B Zwiebach

6.2 Stretched strings

One can also have systems of D-branes with different classes of open strings (open string sectors)



Consider a string stretched between two parallel D24 branes located at $x^{25}_{\circ} = x^{25}_a$ and $x_{\circ} = x^{25}_b$

String endpoints $X^{25}_{ab}(\tau, \sigma=0) = x_a$, $X^{25}_{ab}(\tau, \sigma=\pi) = x_b$

$$X^{25}_{ab} = x_a^{25} + \frac{1}{\pi}\underbrace{(X_b^{25} - X_a^{25})}_{\Delta X_{ab}}\sigma + \sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha_n^{25} e^{-in\tau}\sin(n\sigma)$$

$$\alpha_0^{25} = \frac{1}{\sqrt{2\alpha'}\,\pi}(X_b^{25} - X_a^{25})$$

mass-shell condition: $M^2_{ab} = -p\cdot p = \underbrace{\left(\frac{X_b^{25} - X_a^{25}}{2\pi\alpha'}\right)^2} + \frac{1}{\alpha'}(N-1)$

shift of mass-levels: $\left(\frac{\Delta X}{2\pi\alpha'}\right)^2 = (T\,\Delta X)^2$ = mass^2 of a string stretched between the branes

Spectrum of the stretched string:

· N = 0 $|K, ab\rangle$

information about which D brane string ends live

Chan-Paton labels

labels $[a\ b]$

a denotes brane on which end $\sigma = 0$ lives

b denotes brane on which end $\sigma = \pi$ lives

in our case a, b take values 1 or 2; more generally, for N D branes they take values 1, …, N

(For a string with ends on the same brane $a = b$)

mass shell condition: $$M^2_{ab} = -\frac{1}{\alpha'} + \left(\frac{\Delta X_{ab}}{2\pi\alpha'}\right)^2$$

tachyon if $|\Delta X_{ab}| < 2\pi\sqrt{\alpha'}$

$\cdot \; \underline{N=1}$ $\quad M_{ab}^2 = \left(\frac{\Delta X_{ab}}{2\pi\alpha'}\right)^2$ $\quad M_{ab}^2 = -p\cdot p = \underbrace{\left(\frac{X_b^{25} - X_a^{25}}{2\pi\alpha'}\right)^2} + \frac{1}{\alpha'}(N-1)$

$|S, K, ab\rangle = S \cdot \alpha_{-1} |K, ab\rangle \quad (K\cdot S = 0)$ massive vector on 25 dim space time

$\eta \, \alpha_{-1}^{25} |K, ab\rangle$

null states

$$L_{-1} |K; ab\rangle = K\cdot \alpha_{-1} |K; ab\rangle + \frac{\Delta X_{ab}}{\sqrt{2\alpha'}\,\pi} \alpha_{-1}^{25} |K; ab\rangle$$

no osc

with $K\cdot K = 0$

Longitudinal mode & scalar are null!

null states

$$L_{-1} | K; ab \rangle = \frac{1}{2} \sum_n \alpha_{-1-n} \cdot \alpha_n | \cdots \rangle$$

$$= \frac{1}{2} \left(\alpha_{-1} \cdot \alpha_0 + \sum_{n \geq 0} \left(\cancel{\alpha_{-1-n} \cdot \alpha_n} + \alpha_{-1+n} \cdot \alpha_{-n} \right) \right) | \cdots \rangle$$

$$= \frac{1}{2} \left(\alpha_{-1} \cdot \alpha_0 + \alpha_{-1} \cdot \alpha_0 \right] | \cdots \rangle = (\alpha_{-1} \cdot \alpha_0) | \cdots \rangle$$

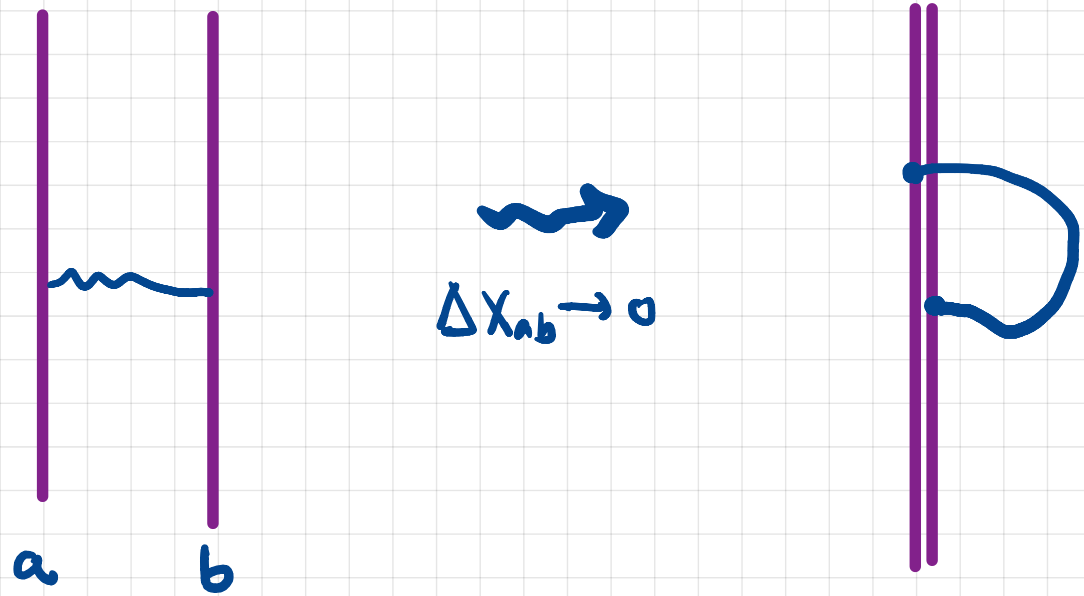
$$= \left(\alpha_{-1} \cdot K + \alpha_{-1}^{25} \alpha_0^{25} \right) | \cdots \rangle$$

$$= K \cdot \alpha_{-1} | \cdots \rangle + \frac{\Delta X}{\sqrt{2\alpha'} \pi} \alpha_{-1}^{25} | \cdots \rangle \qquad \left(\alpha_0^{25} = \frac{1}{\sqrt{2\alpha'} \pi} \Delta X \right)$$

with $K \cdot K = 0$

Coincident limit:

suppose we have N D-branes at X^{25}_a, X^{25}_b, ...



stretched string state between Dbrane at x_a & D-brane at x_b $\underset{\Delta X_{ab}\to 0}{\rightsquigarrow}$ massless gauge fields

N massive vector fields

$|\xi; k, ab\rangle \quad a,b = 1,\dots,N$

N^2 sectors

$\xi\cdot\alpha_{-1}|k; ab\rangle$ + scalars

$a,b = 1,\dots,N$ Chan-Paton labels

all (ab) strings have the same spectrum

One can show that the spectrum has a manifest $U(N)$ symmetry and that these (N^2) states transform in the adjoint representation of $U(N)$

One can choose a basis for these states

$$|\rho, k; A\rangle = \sum_{a,b} (t^A)^a{}_b \, |\rho, k; ab\rangle$$

$A = 1, \dots, N^2$

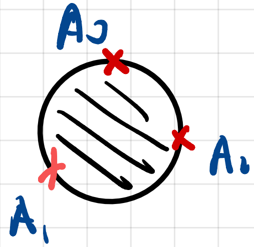
Chan-Paton factors

hermitian basis of $u(N)$

$$\mathrm{tr}(t^A t^B) = \delta^{AB}$$

$$\left(\text{Null states}: \; L_{-1}|0, \underline{k}; ab\rangle = |\underline{k}; \underline{k}; ab\rangle + \frac{\Delta X_{ab}}{\sqrt{2\alpha'}\,\pi} |\eta; \underline{k}; ab\rangle\right)$$

3-point coupling of massless vectors



$$\mathcal{A}(\zeta_1, k_1, A_1;\ \zeta_2, k_2, A_2;\ \zeta_3, k_3, A_3)$$

$$\sim g_o\, \delta(k_1+k_2+k_3)\Big\{ \zeta_1\cdot k_{23}\ \zeta_2\cdot\zeta_3 + \zeta_2\cdot k_{31}\ \zeta_1\cdot\zeta_3 + \zeta_3\cdot k_{12}\ \zeta_1\cdot\zeta_2$$

$$+ \frac{\alpha'}{2}\, \zeta_1\cdot k_{23}\ \zeta_2\cdot k_{31}\ \zeta_3\cdot k_{12} \Big\} \times \mathrm{tr}\left(t^{a_1}[t^{a_2}, t^{a_3}]\right)$$

This is the 3 point vertex operator associated to the action

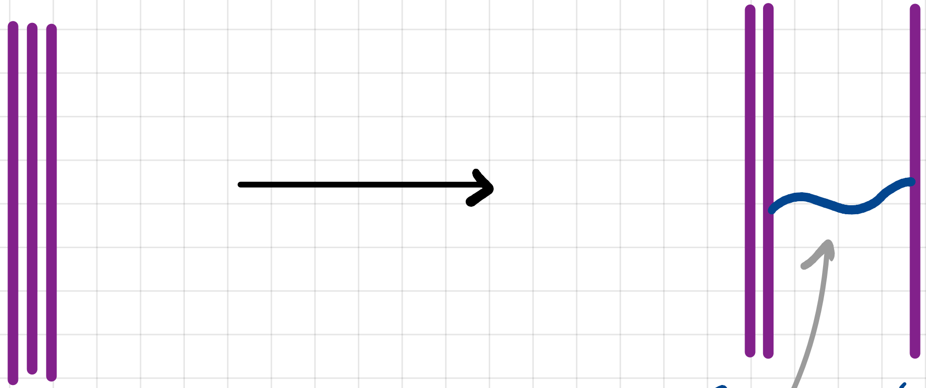
$$\underset{\text{EFT action on D24 brane}}{\mathcal{L}} = \underbrace{-\frac{1}{4}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})}_{\text{Yang-Mills}} - \frac{2i\alpha'}{3}\mathrm{Tr}(F_\mu{}^\nu F_\nu{}^\rho F_\rho{}^\mu) + \text{scalars}$$

← α'-correction

for a $U(N)$ non-abelian gauge theory

One can derive this using β-functions
(needs extra tools: boundary couplings, boundary renormalization flow!)

D-brane picture of non-abelian gauge theory leads to a rather nice picture of the Higgs mechanism.



U(3) U(2) U(1)

stack of 3 Dbranes

$M_{ab}^2 = \left(\frac{|\Delta X|}{2\pi\alpha'}\right)^2$ mass of string stretched between branes

$\Downarrow$

massive W-bosons : $|\xi; \underline{k}; ab\rangle$

null states $|\underline{k}; \underline{k}; ab\rangle + \frac{\Delta X_{ab}}{\sqrt{2\alpha'}\pi} |n; \underline{k}; ab\rangle$

longitudinal W's

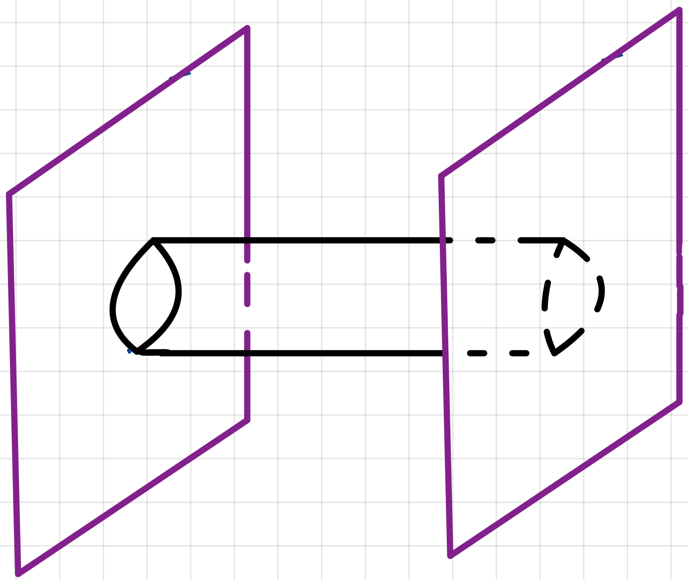
W's "eat" the Higgs

Epilogue: D-branes as dynamical objects?

If they are, maybe they need to be included in the perturbative description of strings? how?

Estimate mass scale relevant to D-branes by computing its tension

closed string exchange between two D-branes



gravitational coupling $\kappa \sim g_0^2$

$$\mathcal{A} \sim \kappa^2 \tau_p^2 \sim (g_0)^0 = 1$$

D-brane tension

$$\tau_p \sim \frac{1}{g_0^2} \Rightarrow$$

D-branes are massive "non perturbative" objects

Polchinski 1995, "duality revolution"

6.3 Final remarks:

We have seen that the theory of quantised string has a very rich structure

- quantised gravity (at low energies we obtain Einstein's gravity)
- gauge fields
- consistency of the theory $\rightsquigarrow$ fixes dimension of spacetime

- compactifications / strings in (nontrivial) background fields

↳ this course: $\mathbb{R}^{1,25}$, $\mathbb{R}^{1,24} \times S^1_{\mathbb{R}}$

More generally:
- $\mathbb{R}^{1,d-1} \times M^{D-d}$ ← M Ricci flat to leading order in α', (from $\beta = 0$)
- $X^{1,d-1} \times M^{D-d}$ ← geometry dictated by $\beta = 0$

  eg max symmetric ↺

  eg $AdS^3 \times S^7$, $AdS^5 \times S^5$, etc ...
- or even more general set ups

- T duality
- emergence of (non-perturbative) p branes

More to learn

- dualities (Mirror symmetry, AdS/CFT)
- emergence of non-perturbative Branes
- CFT & AdS/CFT
- strong coupling regime
- Black hole physics
- realistic phenomenology
- mathematical structures

  ↳ geometry (differential, algebraic, ...)
  topology, number theory, algebra ...

⋮

Improvements:

ST 2

- remove tachyons $\rightsquigarrow$ superstrings
  (fermions in 2dim NLσM : supersymmetric WS theory)
- spacetime fermions
- superstring theory $\rightsquigarrow$ spacetime dim $= 10$

  ⋮
- dualities

End of String Theory I

Thanks!