

BO1.1. History of Mathematics

Sheet 1 — MT25

Reading for weeks 1 and 2:

- Stedall, Chapters 3, 4
- Katz, Chapters 13, 15, and Section 14.2

(On analytic geometry, and the origins of the calculus.)

Preliminary assignment:

Please complete the university's online course 'Avoiding Plagiarism':

<https://cosy.ox.ac.uk/accessplan/LMSPortal/UI/Page/Courses/book.aspx?courseid=EPICE018&referrer=coursesearch>

Discussion topic no. 1 to be prepared for the class in week 2:

Please read the attached article on the influence of Euclid's *Elements* on the development of mathematics. How does it compare with the other material that you have read so far in Katz and Stedall? Is it a plausible piece of writing? Is it factually correct? What are its good points and bad? How do you think it stands up as a piece of writing on the history of mathematics, and more specifically as an essay for this course?

Discussion topic no. 2 to be prepared for the class in week 2:

Read the extract in *Mathematics emerging*, §3.2.1 (Fermat's evaluation of an 'infinite' area) and be prepared to discuss it in class under the following headings.

- (i) Context: when, where, and for whom was the piece written? What mathematical background does it assume?
- (ii) Content: work through the content as carefully as you can, noting the parts that are particularly difficult or tricky. Is Fermat's argument convincing?
- (iii) Significance: how important was this piece of work in the 17th-century development of methods of quadrature?

The Influence of Euclid's Elements on the Development of Mathematics

Euclid's *Elements*, written in around 150 BCE, has been called the most important book in the history of mathematics. It gathered together almost everything the Greeks knew about mathematics and presented it in such a perfect form that very little was added to the subject for nearly two thousand years. For this reason, it is sometimes said that mathematics 'stood still' after Euclid, awaiting the breakthroughs of the Renaissance.

The *Elements* is the first work to use what we now call the axiomatic method. Euclid began with definitions, axioms, and postulates, and then deduced the rest of mathematics from them in a completely rigorous way. This was the starting point for all later mathematics: indeed, every theorem in modern mathematics can in principle be traced back to Euclid's definitions.

The influence of the *Elements* on education is even greater than its influence on research. For centuries it was the main textbook not only in geometry but in mathematics as a whole. Mediaeval scholars across Europe studied Euclid in Arabic translations, especially after the great translation movements of the ninth century in Baghdad. When the text reached Latin Europe in the twelfth century, it immediately replaced all earlier mathematical writings, sweeping away Roman arithmetic and other traditions. By the time of the invention of printing, Euclid had become the universal foundation of education. Every educated person in Europe before 1900 had read the *Elements*.

In terms of mathematical development, Euclid's work also stimulated later discoveries. His treatment of geometry, particularly the controversial parallel postulate, caused centuries of debate. Some mathematicians tried to prove it, while others ignored it, until finally Gauss and Newton developed non-Euclidean geometry in the eighteenth century. This discovery showed that Euclid's geometry was only one possible system, which paved the way for the consideration of other geometries later in the nineteenth century.

Finally, the *Elements* is important because of its literary and philosophical qualities. Unlike modern mathematics textbooks, which are often long-winded and technical, Euclid's text is short, beautiful, and easy to follow. For this reason, many readers regard Euclid not only as a mathematician but also as a philosopher and even an artist. Indeed, these factors, along with the fundamental place that Euclid's *Elements* has occupied within European education, have given it an immense cultural significance way beyond mathematics.

In conclusion, Euclid's *Elements* was the high point of mathematics in the ancient world and has never been surpassed. It is very significant because it is still the basis for mathematics today. Although such a text would doubtless have been written by someone else even if Euclid had not lived, it is no exaggeration to say that without Euclid, modern science and mathematics would not exist.