Mathematical and Theoretical Physics Induction

Lionel Mason, MTP Director of Studies

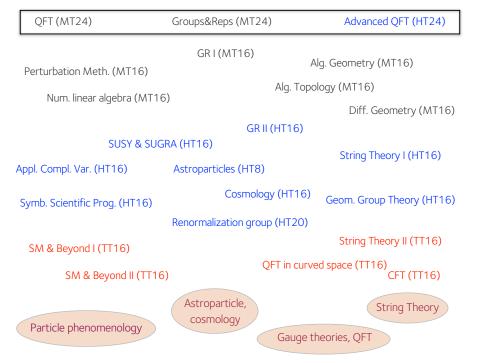
N1.13, The Mathematical Institute, Oxford lmason@maths.ox.ac.uk

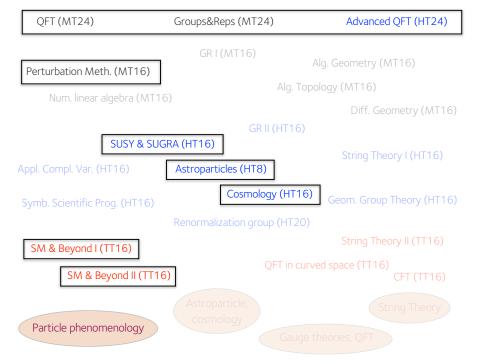
Announcements:

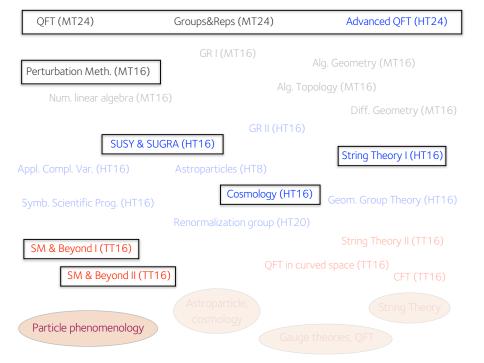
- Q& A / Surgery (course choices etc.): 11.30-12.30am Friday 10th October, Martin Wood, Clarendon Physics (optional).
- ► We need 2 course reps for the JSC, one from MMathPhys, one from MSc. Please volunteer if interested!

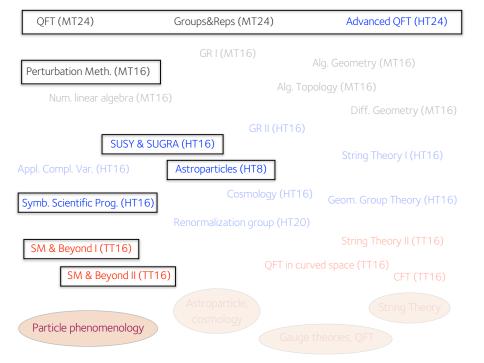
Quantum Field Theory, Particle Physics and String Theory

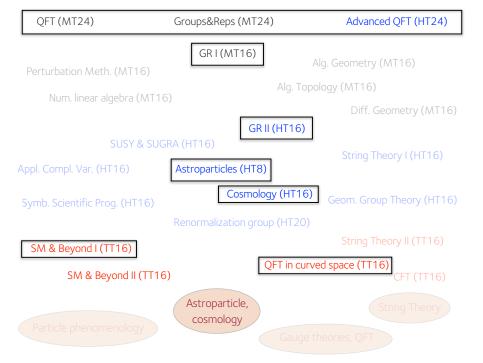
Caution: these are suggestions. Always look at the handbook / discuss with your academic advisor

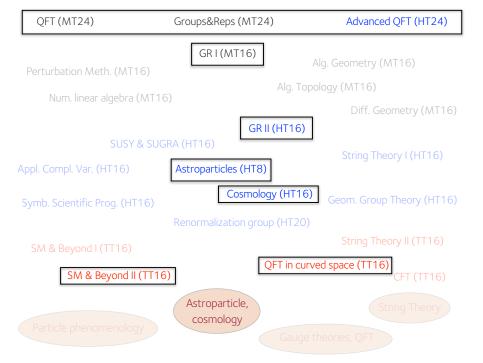


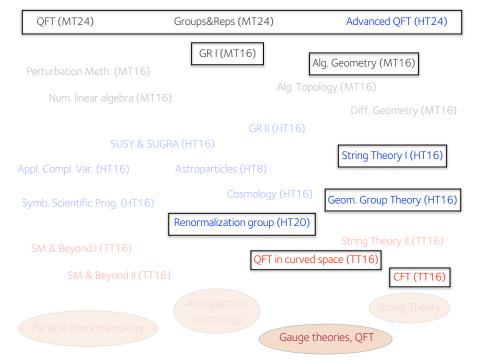


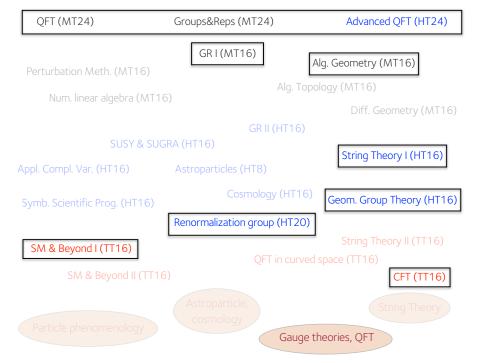


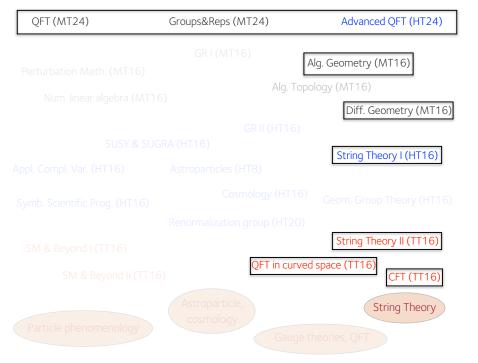


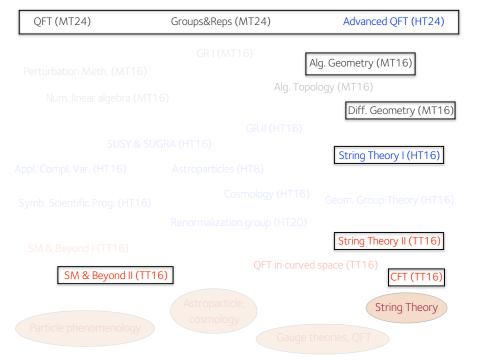












Group activities Seminar

QCD lunch Coffee

Mathematical foundations

Mathematical foundations is not a single pathway:

- Mathematics underpins all parts of physics.
- ▶ Much of mathematics, pure and applied, is used in this process.

If you are coming from physics, if you are interested in

- particle physics and string theory you will benefit from: groups and representations, differential geometry, algebraic geometry,
- fluids, condensed matter or plasmas, you will benefit from perturbation methods, complex variables, numerical linear algebra, scientific computing.

If you are coming from Mathematics with an interest in

- geometry and topology, this underpins general relativity, string theory compactifications, AdS/CFT and supersymmetric systems.
- group theory and algebra underpin all quantum systems from condensed matter to particle physics and quantum information.
- analysis, both pure and applied, underpins all differential equations from fluids through to GR and quantum theory.

Most parts of theoretical physics can be studied from a mathematical perspective.

Mathematical Foundations pathways

Pathway	MT	НТ	TT
Fluid Dynamicist "CONTINUA" Core 4.5 units Total 10 - 10.75 units	1. Kinetic Theory 28 2. Pert. Methods 16	1. Adv. Fluid Dyn. 16 2. Three of Soft Matter Phys. 16 Collisionless Plasmas 18 Geophysical Fluids 16 Complex Variables 16	1-2 <i>Two of</i> Collisional Plasmas 18 Astrophysical Gas Dyn. 20 Dissertation
Applied Mathematician "APPLICATA" Core 5.5–7.25 units Total 10–11 units	1-2. Two of Noneq. Stat. Phys. 24 Kinetic Theory 28 GR I 16 3. Pert. Methods 16 4. One of Diff. Geometry 16 Num. Lin. Algebra 16	1. Adv. Fluid Dyn. 16 2. One of Geophysical Fluids 16 Networks 16 Collisionless Plasmas 18 Galactic Dyn. 16 GR II 16 3. Complex Variables 16	1-2. Two of Collisional Plasmas 18 Astrophysical Gas Dyn. 20 (10HT, 10 TT) Dissertation

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Pathway	MT	HT	TT
Mathematical physicist "GEOMETRA" Core 5.5 units Total 10–10.5 units	1. QFT 24 2. GR I 16 3. Diff. Geometry 16 4. One of Groups & Repr. 24 Algebraic Topology 16 Algebraic Geometry 16 Topological Field Th. 16	1. String Theory I 16 2. One of Advanced QFT 24 SUSY & SUGRA 16 GR II 16 Random Matrix Theory 16 Quantum Information 16	1. String Theory II 16 2. Two of CFT 16 Conformal Field theory 16 The Standard Model 16* Beyond the SM 16 QFT in Curved Space 16*
Maths of Gauge theories and QFT "PARTICULATA" Core 5.5 units Total 10 units	1. QFT 24 2. Groups & Repr. 24 3. One of GR I 16 Differentiable Manifolds 16	1. Advanced QFT 24 2. SUSY & SUGRA 16 3. Two of String Theory I 16 GR II 16 Riemannian Geometry 16	Two of String Theory II 16 Renormalization group 16 Conformal Field Theory 16 QFT in Curved Space 16
Hard-core String Theorist "SUPERCORDULA" Core 7.5 units Total 11.5 units	1. QFT 24 2. Groups & Repr. 24 3. One of GR I 16 Diff. Manifolds 16 Algebraic Geometry 16	1. Advanced QFT 24 2. String Theory I 16 3. One of SUSY & SUGRA 16 Riemannian Geometry 16 Low Dimensional Topo 16 GR II 16 Cosmology 16	1. String Theory II 16 2. CFT 16 3. One of The SM and Beyond I 16 The SM and Beyond II 16 QFT in Curved Space 16

	Overview of Lecture Courses				
	Theoretical Particle Physics	Theoretical Condensed Matter Physics	Astrophysics, Plasma Physics & Physics of		
\vdash	Quantum Field T	heory (P) (24)	Continuous Media		
	,	Field Theories and Collective Phenomena in Condensed Matter (P U.C., (P) (24)			
		Kinetic Theory (P) (28)			
			Anyons & Topological Quantum Field Theory (P)		
			Quantum Processes in Hot Plasma		
МТ	General Relativity I(MU,C7.5) (M) (16)		General Relativity I(MU,C7.5) (M)		
MI	Perturbation Methods MUC5.5) (M) (16)				
	Numerical Linear Algebra MUCG-1) (M) (16)				
Groups and Representations (P) (24)		sentations (P) (24)			
	Algebraic Topology (MU.C3-1) (M) (16)				
	Differentiable Manifolds (MU.C3.3) (M) (16)		Differentiable Manifolds (MU,C3.3)		
	Advanced Philosophy of Physics (Phil.) (24)				
	Algebraic Geometry (MU.C3.4) (M) (16)				
		Advanced Fluid Dynamics (P) (16) unnian Geometry MU.C.,111 (M) (16) Nonequilibrium Statistical Physics (P) (16)			
	Riemannian Geometry (MU.C3.11) (M) (16)				
	Advanced QFT (P) (24)	Quantum Matter (P) (16)	High Energy Density Physics (P) (16)		
	String Theory I (M) (16)		Collisionless Plasma Physics (P) (18)		

Geophysical Fluid Dynamics (P) (16)

Supersymmetry & Supergravity (M) (16)

MMath part C courses not on MTP list

The list below is out of date!

For up-to-date list browse link to part C at https://courses.maths.ox.ac.uk/ .

Michaelmas	Hilary	
Model Theory	Godel's Incompleteness Theorem	
Analytic Topology	Axiomatic Set Theory	
Lie Algebras	Representation Theory of Lie Algebras	
Homological Algebra	Infinite Groups	
Category Theory	Non-Commutative Rings	
Elliptic Curves	Introduction to Schemes	
Functional Analysis	Lie Groups	
Functional Analytic Methods for PDEs	Probabilistic Combinatorics	
Complex Analysis: Conformal Maps & Geometry	Analytic Number Theory	
Solid Mechanics	Computational Algebraic Topology	
Topics in Fluid Mechanics	Linear Operators	
Mathematical Geoscience	Fixed Point Methods for Nonlinear PDEs	
Mathematical Physiology	Elasticity and Plasticity	
Approximation of Functions	Mathematical Mechanical Biology	
Stochastic Differential Equations	Continuous Optimisation	
Combinatorics	Finite Element Method for PDEs	
	Stochastic Analysis and PDEs	

Up to 3 units can be taken from the part C's of both MMath and MPhys. Authorization from director of studies is required by week 4 MT.

If you dont see a dissertation topic in the handbook that is right for you, ask around!

Where does it lead in Oxford Maths?

The Mathematical Physics group has a history of deep two-way interactions with geometry, algebra and analysis both via relativity and QFT.

- Scattering amplitudes via twistors, galois theory of periods or AdS/CFT.
- Holography, including Celestial and Carrollian and in quantum gravity.
- ► The geometry and number theory of string compactifications and associated exact computations in quantum supersymmetric gauge theories and AdS/CFT.
- ► Mathematical structures in QFT, generalized and categorical symmetries, etc..
- Quantum information/foundations.

In other groups:

- ▶ Homological mirror symmetry in symplectic and algebraic geometry.
- ▶ Topological quantum field theories in geometry, topology (and computer science).
- Fluids, plasmas, geophysics and Mathematical Biology in OCIAM.
- ▶ The differential equations of mathematical physics in OXPDE.
- ▶ Stochastic quantization in the probability and analysis group.

and many more.

Oxford Master Course in

Mathematical and Theoretical Physics

Caroline Terquem, Chair of JSC (Joint Supervisory Committee)

Rudolf Peierls Centre for Theoretical Physics, Beecroft Building

Department of Physics/Theoretical Physics

Mathematical Institute

What is it?

A high-level master course in Mathematical and Theoretical Physics

MMathPhys (Students with Oxford BA): you pursue in your fourth year Instead of following the fourth year of the MPhys/MMath/MPhysPhil.

OR

MSc (Students with BA from elsewhere): you are admitted to a one year program from a prior degree elsewhere.

Key contacts

- course website: http://mmathphys.physics.ox.ac.uk
- email: mmathphys@maths.ox.ac.uk
- course administration: Eleanor Kowol
- your departmental academic advisor
- director of study: Lionel Mason
- chair of JSC: Caroline Terquem
- your college academic advisor/senior tutor

We also have a facebook page, students have whatsapp chat, there have been discord servers, slack channels etc, in the past too

Key documents

https://mmathphys.physics.ox.ac.uk/students

- course handbook
- exam conventions
- past exam papers
- dissertation guidance

Yes.....

It will be exciting

It will be challenging

It will be a lot of work

... we are confident you can do it!

What are you required to do?

Ten units (1 unit=16h):

- a) at least 4 units assessed by written invigilated exams
- b) at least 3 further units assessed by written invigilated exams or other formal assessment
- c) at least 3 other units (with homework completion only or formal assessment)

(Formal assessment = written invigilated exam, take-home exam, mini-project)

Dissertation: replaces one or two units in (b) or (c)

- It is not required that you do original research, but many students do
- Start exploring options NOW! (first deadline is Monday week 6 this term)
- There is a list of "suggested topics". But many more topics are possible.

You are free to choose your own adventure... ...But...

Be Responsible!

- Do not put all your work at the end of the year!
- Pay attention to the exam schedule (some courses from MT are examined in TT)

And keep in mind:

- Many of the courses go very fast.
- You may have to do more learning on your own (... and a lot over the breaks)
- You may have to catch up on background info.
- The exercises you are assigned to hand in may not really be enough.

What do we learn?

The four main areas covered by the course are:

- Theoretical Condensed Matter Physics
- Theoretical Astrophysics, Plasma Physics and Physics of Continuous Media
- Quantum Field Theory, Particle Physics and String Theory
- Mathematical Foundations of Theoretical Physics

Oxford is very strong in plasma physics, astrophysics and fluids, across 3 sub-departments: Theoretical Physics, Astrophysics, AOPP (Atmospheric, Oceanic and Planetary Physics plus the Rutherford Appleton Laboratory

- Kinetic Theory (MT) of gases, plasmas and self-gravitating systems
- Advanced Fluid Dynamics (HT): non-Newtonian fluids and MHD

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Thermodynamics is a **phenomenological** theory of matter which is derived from experiments.

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Fluid Dynamics is the limit of kinetic theory when the mean free path is small compared to other characteristic lengthscales.

ASTROPHYSICS

PLASMA PHYSICS

Kinetic Theory

gas kinetics

gravitational kinetics

plasma kinetics

Advanced Fluid Dynamics

non-Newtonian fluids

magnetohydrodynamics

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----- Non-equilibrium Statistical Physics

Advanced Fluid Dynamics

non-Newtonian fluids magnetohydrodynamics

Soft matter (liquid crystals, polymers, biological materials...)

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Geophysical
Fluid
Dynamics

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PLASMA PHYSICS

Kinetic Theory

gas kinetics

gravitational kinetics

plasma kinetics

Non-equilibrium Statistical Physics

Advanced Fluid Dynamics

non-Newtonian fluids

magnetohydrodynamics

Soft matter (liquid crystals, polymers, biological materials...)

Geophysical Fluid Dynamics

Galactic & Planetary
Dynamics

Astroparticle Physics

Cosmology

GR, particle theory....

ASTROPHYSICS

PLASMA PHYSICS

Kinetic Theory

gas kinetics

gravitational kinetics

plasma kinetics

Non-equilibrium Statistical Physics

Advanced Fluid Dynamics

non-Newtonian fluids

magnetohydrodynamics

Soft matter (liquid crystals, polymers, biological materials...)

Geophysical
Fluid
Dynamics

Galactic & Planetary
Dynamics

Astroparticle Physics

Cosmology

Collisionless

Plasma

Physics

Collisional

Plasma

Physics

GR, particle theory...

What is condensed matter?

- Condensed Matter ≈ The study of complex systems (both classical and quantum) with many components (many parts, many atoms, many...).
 - Often meaning "substances" or the physics of "stuff"
 i.e., the world around you!
- Largest subfield of physics
 - 1/3 of all physics as measured by number of practitioners
- Extremely Broad and Diverse field

Example: Meeting of American Physical Society Condensed Matter (over 10,000 physicists attend...)

First day of the meeting at 8am lectures on the topics of...

Superconductors,

Superfluids,

Glasses,

Polymers,

Microfluidics

Crystal Growth Kinetics

Spintronics

Phase Transitions

Quantum Criticality

Bose Condensates

Ultra High Pressure

Ultra Low Temperature

Ultra Fast Physics

Topological Matter

Quantum Computation

Quantum Algorithms

Ferromagnetism

Anti Ferromagnetism

Heavy Fermions

Multiferroics

Liquid Crystals

Graphene

Interfaces

Heterostructures

Fractionalized Charges

Beyond Bosons and Fermions

Quantized Hall Effects....

Phase Transitions

... Plus 40 other topics. .. And this all at 8am the first day.

At 11am there are 65 completely different topics being discussed!

... and this goes on all week long!

Strong Overlaps Between Condensed Matter and....

- Chemistry
- Material Science
- Biology
- Atomic Physics
- High Energy Physics (FIELD THEORIES!!!)
- Nanoscience
- Quantum Information Sciences
- Topology
- And increasingly... String Theory, Black Hole Physics (AdS/CFT)

Condensed Matter is good for your career:

- LOTS of experimental data to study
 - Experiments are quick and cheap
- Explain the world around you!
- Many job opportunities in condensed matter
 - Academic department of math, physics, biology, chemistry, materials, engineering.
 - National Labs
 - Industry!!!

(Condensed matter enables technological revolutions!)

WHAT IS CONDENSED MATTER THEORY?

For a theorist key tool is **field theory** (just as in high energy), **but**

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High energy = reductionist
= smaller and smaller = higher and higher energy
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Condensed matter = anti-reductionist = how do pieces act together and what new physics can **emerge** when this happens

Extremely intellectually challenging and mathematically sophisticated:

We are not interested in the "easy" asymptotically free regime at high energy (i.e., accelerator physics), but rather the complex highly interacting physics of low energy (i.e., everything else).

Emergent phenomena: that you would never guess can arise from the "microscopics." Ex: can a particle with 1/3 of an electron charge can arise from a many electron quantum soup? (it does!)

BROKEN SYMMETRIES AND THE MASSES OF GAUGE BOSONS

Peter W. Higgs

Tait Institute of Mathematical Physics, University of Edinburgh, Edinburgh, Scotland (Received 31 August 1964)

In a recent note¹ it was shown that the Goldstone theorem,² that Lorentz-covariant field theories in which spontaneous breakdown of symmetry under an internal Lie group occurs contain zero-mass particles, fails if and only if the conserved currents associated with the internal group are coupled to gauge fields. The purpose of the present note is to report that, as a consequence of this coupling, the spin-one quanta of some of the gauge fields acquire mass; the longitudinal degrees of freedom of these particles (which would be absent if their mass were zero) go over into the Goldstone bosons when the

coupling tends to zero. This phenomenon is just the relativistic analog of the plasmon phenomenon to which Anderson³ has drawn attention: that the scalar zero-mass excitations of a superconducting neutral Fermi gas become longitudinal plasmon modes of finite mass when the gas is charged.

about the "vacuum" solution $\varphi_1(x) = 0$, $\varphi_2(x) = \varphi_0$:

$$\partial^{\mu} \{ \partial_{\mu} (\Delta \varphi_1) - e \varphi_0 A_{\mu} \} = 0, \qquad (2a)$$

$$\{\partial^2 - 4\varphi_0^2 V''(\varphi_0^2)\}(\Delta \varphi_2) = 0,$$
 (2b)

$$\partial_{\nu} F^{\mu\nu} = e \varphi_0 \{ \partial^{\mu} (\Delta \varphi_1) - e \varphi_0 A_{\mu} \}. \tag{2c}$$

Equation (2b) describes waves whose quanta have (bare) mass $2\varphi_0\{V''(\varphi_0^2)\}^{1/2}$; Eqs. (2a) and (2c) may be transformed, by the introduction of new variables

$$B_{\mu} = A_{\mu} - (e \varphi_0)^{-1} \partial_{\mu} (\Delta \varphi_1),$$

$$G_{\mu\nu} = \partial_{\mu} B_{\nu} - \partial_{\nu} B_{\mu} = F_{\mu\nu},$$
(3)

into the form

$$\partial_{\mu} R^{\mu} = 0 \quad \partial_{\mu} G^{\mu\nu} + e^{2} G^{\mu} R^{\mu} = 0. \tag{4}$$



The Quantum Matter Sequence:

Quantum Matter 1: Phases of Matter and Field Theories Simon (2nd half MT). Landau theory of phases and phase transitions, Quantum Field Theory, Quantum Many Body Physics. (Exam TT)

Quantum Matter 2: Quantum Fluids Parameswaran. (1st part of HT). Superfluids, Superconductors, Fermi Liquids, Quantum Hall Physics. (Exam or HW)

Quantum Matter 3: Quantum Dynamics and Information in Many Particle Systems Essler (End of HT-Start of TT) Entanglement of Many Body Systems,
Spin Chains, Quantum Quenches, Eigenstate thermalization, Open and driven systems. (Exam or HW)

Quantum Matter 4: Renormalization and Bosonization Sondhi (end of TT) Quantum Field theory in the nonperturbative regime

To get you up to speed at the forefront on quantum condensed matter theory!

Also We Encourage!....

Anyons and Topological Quantum Field Theories Simon (1st half MT).

Particles that are neither bosons nor fermions. Relations to condensed matter, quantum information, high energy, topology. (Exam HTO or HW)

[For CondMat, High Energy, Maths--- everyone can enjoy!]

For Quantum Condensed Matter:

Quantum Field Theory (MT) 1.5 units

Algorithms and Computations in Theoretical Physics (HT)

C7.4 Intro to Quantum Information (HT)

C7.7 Random Matrix Theory (HT)

Conformal Field Theory (TT)

For Non-Quantum Condensed Matter

C5.4 Networks (HT)

Nonequilibrium Statistical Physics (HT)

Topics in Soft and Active Matter Physics (TT)