Fridays@*n*, *n*=*11* or 2

WEEK 3 @11
What does a good maths solution look like?

WEEK 4 @ 2
Looking and applying for jobs

WEEK 5 @ 11
How to make the most of your tutorials and lectures

WEEK 6 @ 2
What's it like doing a PhD in maths?

WEEK 7 @ 11
How to manage your time effectively

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What does a good maths solution look like?

31 October 2025

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Today

- We'll share the slides with you afterwards.
- You might like to have a pen/pencil ready anyway.

Writing maths

arXiv > math > arXiv:1110.5008

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Mathematics > Group Theory

[Submitted on 22 Oct 2011 (v1), last revised 25 Oct 2011 (this version, v2)]

The structure of approximate groups

Emmanuel Breuillard, Ben Green, Terence Tao

Let K >= 1 be a parameter. A K-approximate group is a finite set A in a (local) group which contains the identity, is symmetric, and such that A^2 is covered by K left translates of A.
The main result of this paper is a qualitative description of approximate groups as being essentially finite-by-nilpotent, answering a conjecture of H. Helfgott and E. Lindenstrauss. This may be viewed as a generalisation of the Freiman-Ruzsa theorem on sets of small doubling in the integers to arbitrary groups.
We begin by establishing a correspondence principle between approximate groups and locally compact (local) groups that allows us to recover many results recently established in a fundamental paper of Hrushovski. In particular we establish that approximate groups can be approximately modeled by Lie groups.
To prove our main theorem we apply some additional arguments essentially due to Gleason. These arose in the solution of Hilbert's fifth problem in the 1950s.
Applications of our main theorem include a finitary refinement of Gromov's theorem, as well as a generalized Margulis lemma conjectured by Gromov and a result on the virtual nilpotence of the fundamental group of Ricci almost nonnegatively curved manifolds.

Comments: 91 pages
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Questions…

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1. Let A, B, C be subsets of a set X. Write out a proof that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

2. Find the square roots of $-7 + 24i$.

Solutions…

We're going to look at some **sample solutions**.

They are not based on any individual student's work.

For each attempt, please think about what feedback you would give the student.

- What have they done well?
- What could they improve?
- What features would you copy for your own work?

Student A

(A) $a \in A \cap (B \cup C)$

$\Leftrightarrow a \in A$ and $a \in B \cup C$

$\Leftrightarrow a \in A$ and $a \in B$ or $a \in C$

$\Leftrightarrow a \in A \cap B$ or $a \in A \cap C$

$\Leftrightarrow a \in (A \cap B) \cup (A \cap C)$

Student B

(B) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

Suppose $x \in A \cap (B \cup C)$. Then $x \in A$ and $x \in B \cup C$, so x is an element of A and x is an element of B or x is an element of C. So $x \in A \cap B$ or $x \in A \cap C$ or both. So $x \in (A \cap B) \cup (A \cap C)$. So $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$.

Conversely, let $x \in (A \cap B) \cup (A \cap C)$. Then either $x \in A \cap B$ or $x \in A \cap C$ or both. So x is in A and x is in B or x is in C. So $x \in A \cap (B \cup C)$. So $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$.

So each is a subset of the other, so $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

Student C

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(C) If $a \in LHS$ then a is in A and in $B \cup C$ so is in A, and B or C so is in $A \cap B$ or $A \cap C$ so is in RHS.

Student D

(D) __Claim__ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

__Proof__ $\subseteq$: Take $x \in A \cap (B \cup C)$.

Then $x \in A$ and $x \in B \cup C$.

So $x \in A$ and ($x \in B$ or $x \in C$).

If $x \in B$, then since also $x \in A$ have $x \in A \cap B$
so $x \in (A \cap B) \cup (A \cap C)$.

If $x \in C$, then since also $x \in A$ have $x \in A \cap C$
so $x \in (A \cap B) \cup (A \cap C)$.

So $x \in (A \cap B) \cup (A \cap C)$.

$\supseteq$: Take $x \in (A \cap B) \cup (A \cap C)$.

Then $x \in A \cap B$ or $x \in A \cap C$.

If $x \in A \cap B$ then $x \in A$ and $x \in B$,
so $x \in A$ and $x \in B \cup C$, so $x \in A \cap (B \cup C)$.

If $x \in A \cap C$ then similarly $x \in A \cap (B \cup C)$.

Either way, we see $x \in A \cap (B \cup C)$.

Student E

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(E) Write $-7+24i$ in modulus-argument form:

have $|-7+24i| = \sqrt{7^2+24^2} = 25$

and if $\arg(-7+24i) = \theta$ then $\tan\theta = -\frac{24}{7}$.

Let $w = Re^{i\phi}$ be a square root of $-7+24i$.

Then $w^2 = R^2e^{2i\phi} = -7+24i = 25e^{i\theta}$ where $\tan\theta = -\frac{24}{7}$.

So $R=5$ and $-\frac{24}{7} = \tan\theta = \tan(2\phi) = \frac{2\tan\phi}{1-\tan^2\phi}$.

Then writing $t = \tan\phi$ have $12t^2 - 7t - 12 = 0$,

so $t = \tan\phi = \frac{7 \pm \sqrt{7^2+576}}{24} = \frac{7 \pm 25}{24}$.

So $\tan\phi = \frac{4}{3}$ or $\tan\phi = -\frac{3}{4}$.

Since also $R=5$, have $w \in \{3+4i, -3-4i, 4-3i, -4+3i\}$.

But $(4-3i)^2 = 7-24i$ — no good,
$\quad\;\; \overset{\shortparallel}{(-4+3i)^2}$

whereas $[\pm(3+4i)]^2 = -7+24i$, so the square roots are $\pm(3+4i)$.

Student F

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(F)

$(re^{i\theta})^2 = -7+24i$

$r^2 e^{2i\theta} = -7+24i$

$r^2 = |-7+24i| = \sqrt{49+576} = 25 \Rightarrow r=5$

$\tan(2\theta) = \frac{-24}{7}$ $\qquad \theta = \frac{1}{2}\arctan\left(-\frac{24}{7}\right)$

$= \frac{2\tan\theta}{1-\tan^2\theta}$

$7\tan\theta = -12(1-\tan^2\theta)$

$12\tan^2\theta - 7\tan\theta - 12 = 0$

$\tan\theta = \frac{7\pm\sqrt{49+576}}{24} = \frac{7\pm 25}{24} = \frac{4}{3}$ or $\frac{-3}{4}$

$z = 3+4i,\ -3-4i,\ 4-3i,\ -4+3i$

?

Student G

(G)

$$|-7+24i| = \sqrt{7^2+24^2} = 25$$

$$\arg(-7+24i) = \arctan\left(-\frac{24}{7}\right)$$

Modulus-argument form $= 25 \cdot e^{i\arctan\left(-\frac{24}{7}\right)}$

$$\Rightarrow \quad w = \sqrt{25}\cdot e^{i\theta}, \qquad 2\theta = \arctan\left(-\frac{24}{7}\right) + 2\pi k$$

$$w = \pm(3+4i).$$

$$\tan(2\theta) = -\frac{24}{7}$$

$$\tan\theta = t$$

$$\frac{2t}{1-t^2} = -\frac{24}{7}$$

$$12t^2 - 7t - 12 = 0$$

$$t = \frac{7 \pm \sqrt{7^2+576}}{24} = \frac{7\pm 25}{24}$$

$$\tan\theta = \frac{4}{3} \quad \text{or} \quad \tan\theta = \frac{-3}{4}$$

Show the structure of your argument clearly

- Aspire to more than just “not false”.
- Present a coherent logical argument.
- State what you’re going to prove (and label it).
- If you’re doing two directions, then label them.
- If you’re checking properties, then label them.
- Be clear where one idea ends and the next begins.

Words **and** symbols

- Write in sentences with logical flow.
- Read your work out loud.
- Take care with commas, which can be ambiguous.
- "If … then …" is underrated.
- It's fine to reuse words ("so", "then").
- Be very careful with $\Rightarrow$ etc. Read $\Rightarrow$ as "implies".
- Introduce notation explicitly.

Look at your own work critically

- Don't spend ages rewriting in your best handwriting, but you might need to work in rough then write up your ideas
- If possible, then leave some time and look again
- Review what is on the page, not what you had in mind when you wrote it
- Have you proved what you set out to prove?
- Have you justified each step clearly?

Draw on feedback

- Use feedback from your tutors.
- Ask for advice if you’re uncertain whether you’re writing too much/too little/not in the best style.
- With practice, writing well is a habit.
- Everyone can write maths well.

More advice

- The book *How to think like a mathematician* by Kevin Houston has good advice. Chapters 3 and 4, which are about writing maths, are available at http://www.kevinhouston.net/pdf/htwm.pdf
- The book *How to study for a Mathematics degree* by Lara Alcock also has good advice, including a chapter on writing mathematics https://ebookcentral.proquest.com/lib/oxford/detail.action?docID=1073506