Extract from a letter from E. E. Kummer to Joseph Liouville, published along with a note by Liouville in *Journal de mathématiques pures et appliquées* 12 (1847), 136 (also in *Collected papers*, André Weil, ed., 2 vols., Springer, 1975, vol. 1, p. 298).

Breslau, 28 April 1847

At the request of my friend Mr. Lejeune-Dirichlet, I take the liberty of sending you a few copies of a dissertation I wrote three years ago on the occasion of the centenary of the University of Königsberg, and of another dissertation by one of my friends and students, Mr. Kronecker, a distinguished young geometer. In these memoirs, which I beg you to accept as a token of my profound esteem, you will find discussions on some points of the theory of complex numbers composed of the roots of unity, that is to say, of the equation $r^n = 1$, which were recently the subject of some discussion within your illustrious Academy, on the occasion of Mr. Lamé's attempt at a proof of Fermat's Last Theorem. As for the elementary proposition concerning these complex numbers, *that a composite complex number can only be decomposed into prime factors in one way*, which you quite rightly regret in this demonstration that is also flawed in several other respects, I can assure you that *it does not generally hold true* as long as we are dealing with complex numbers of the form $\alpha_0 + \alpha_1 r + \alpha_2 r^2 + \cdots + \alpha_{n-1} r^{n-1}$, but that it can be salvaged by introducing a new kind of complex number that I have called an *ideal complex number*. The results of my research on this subject have been communicated to the Berlin Academy and printed in the *Comptes rendus* (March 1846); a memoir on the same topic will soon appear in Mr. Crelle's Journal. The applications of this theory to the proof of Fermat's Last Theorem have occupied me for a long time, and I have succeeded in making the impossibility of the equation $x^n - y^n = z^n$ dependent on two properties of the prime number n, so that it only remains to determine whether these properties are possessed by all prime numbers. In the event that these results seem to you to be worthy of some attention, you will find them presented in the *Compte rendu* of the Berlin Academy this month.

NOTE BY MR. LIOUVILLE. Mr. Kummer's Memoir, which is mentioned first of all in this Letter and is dated 1844, is written in Latin under the title: *De numeris complexis qui radicibus unitatis et numeris integris realibus constant*. The work by Mr. Kronecker, which follows it and is entitled *De unitatibus complexis*, deals specifically with the complex divisors of the number 1; it appeared in 1845, and the author announces that he will take up the question again in greater detail in Mr. Crelle's Journal. Mr. Kummer's Memoir, being of great interest and not appearing to have been known in France until now, will be published in its entirety in a forthcoming issue, following a work by Mr. Lamé on the same subject, which has been in print for some time. Readers should also consult Mr. Crelle's Journal, the *Comptes rendus* of the Berlin Academy, and finally those of our Academy of Sciences, which contain extensive research by Mr. Cauchy. We need not examine here where the authors we cite agree or differ, nor what the rights of each are to priority of this or that discovery. It is up to time to determine the value of their work and to put everything in its proper place.

Translated by CHRISTOPHER D. HOLLINGS