BO1.1. History of Mathematics
Lecture XVI
Concluding miscellany

MT25 Week 8

Summary

- Summary of the course
- The exam (briefly)
- The history of the history of mathematics*
- Hilary Term reading course

*Non-examinable

Summary of the course: lecture I

Background material (Antiquity onwards):

- How to organise the history of mathematics: by period, culture, people, topics, sources, institutions, ...
- The ancient origins of mathematics, particularly Ancient Greece: Pythagoras, Euclid, Archimedes, Apollonius, Diophantus, ...
- A hugely influential text: Euclid's *Elements*

Summary of the course: lecture II

Background material (Antiquity to the 16th century):

- Preservation of ancient knowledge: manuscripts preserved in Constantinople, Boethius, Islamic scholars, ...
- Transmission of ancient knowledge to early-Renaissance Europe: translations, often via the Middle East
- Transmission histories of Euclid's *Elements*, and the works of both Archimedes and Apollonius
- 16th-century developments: global exploration, international commerce, new technologies, ...
- Influential early figures: Simon Stevin, Thomas Harriot, John Napier, ...

Summary of the course: lecture III

New notation, leading to a new approach to geometry (17th century):

- The advent of symbolic notation: Viète, Harriot, Descartes, ...
- Analytic geometry: Viète and Descartes
- Tangent methods: Descartes, Fermat

Summary of the course: lecture IV

Methods for finding areas, and for handling infinitely small quantities (17th century):

- Quadrature: Archimedes and the circle, Fermat and Brouncker and the hyperbola
- Indivisibles and infinitesimals: Toricelli, Wallis
- Simple 'integrals': Wallis' 'induction', Newton's 'extrapolation'
- The calculus(es) of Newton and Leibniz

Summary of the course: lecture V

- Newton
- The *Principia* and mathematical understanding of the physical world: Kepler's laws, centripetal force, the inverse square law, Newton's laws of motion

Summary of the course: lecture VI

Consolidation of the calculus (17th–18th centuries):

- Publication (or not) of the calculus: Newton and Leibniz
- Challenge problems: isochrone, brachistochrone, ...
- Changing conceptions of a function: Bernoulli, the wave equation, d'Alembert, Euler, another Bernoulli
- Criticisms of the calculus: Hobbes, Berkeley
- Responses to the difficulties: l'Hôpital, Maclaurin, Euler, Landen, Lagrange
- Professionalisation of mathematics

Summary of the course: lecture VII

Infinite series (17th–18th centuries):

- Newton and the general binomial theorem: ‘extrapolation’ and ‘interpolation’
- Newton’s other series: sin, tan, log, ...
- Other early uses of power series: Brouncker, Mercator, Wallis, Gregory, ...
- Calculus and power series combined: Taylor and Maclaurin series
- Lagrange’s use of power series

Summary of the course: lecture VIII

New concepts, new rigour (mainly 19th century):

- Fourier series
- Cauchy sequences: Bolzano, Cauchy, Abel, ...
- Continuity: Wallis, Euler, Bolzano, Cauchy
- Limits: Wallis, Newton, d'Alembert, Cauchy
- Differentiability: Leibniz, Lagrange, Ampère, Cauchy, ...
- Weierstrass and modern rigour: ε and δ
- French and German institutions

Summary of the course: lecture IX

Algebra as equation solving, and the theory of equations (Antiquity to the 18th century):

- Quadratic equations: Babylon, al-Khwārizmī, Fibonacci
- Cubic equations: del Ferro, Tartaglia, Cardano
- Quartic equations: Ferrari, Cardano
- 16th-century developments: Bombelli, Stevin, Viète
- 17th-century developments: Harriot, Descartes, Hudde, Tschirnhaus, Newton
- 18th-century speculations about the resolvent: Euler, Bézout

Summary of the course: lecture X

Galois theory, and abstract algebra (18th–19th centuries):

- Lagrange's reflections on resolvents
- The number of 'values' a function may take: Ruffini, Cauchy, Abel
- Galois and his groups
- Cauchy and his groups
- Cayley and his groups
- Weber's axioms
- The advent of abstract algebra: rings, ideals, integral domains, fields, ...

Summary of the course: lecture XI

More new concepts, and further ideas about rigour (19th century):

- Uniform convergence: Cauchy, Weierstrass, Stokes, Seidel
- Integration: as an area, and by anti-differentiation
- Definite integrals: Fourier, Cauchy
- The Fundamental Theorem of Calculus
- Riemann and Lebesgue integrals

Summary of the course: lecture XII

New definitions of real numbers, and sets (19th century):

- The need for completeness: Bolzano, Cauchy, Weierstrass
- Dedekind cuts, and other approaches to real numbers
- Sets

Summary of the course: lecture XIII

Understanding complex numbers, and using them in analysis (16th–19th centuries):

- Complex numbers: Cardano, Bombelli, Harriot, Wallis, Bernoulli, Euler, Gauss, Wessel, Argand, ...
- Substitution of complex variables for real ones: Bernoulli, Euler, Laplace, Poisson, Cauchy
- Cauchy and complex analysis: the Cauchy-Riemann equations, contour integration, Cauchy's Integral Theorem, Cauchy's Integral Formulae, Cauchy's Residue Theorem, ...
- Riemann's contributions
- 'Analysis'

Summary of the course: lecture XIV

Separate concepts brought together under the heading of 'linear algebra' (Antiquity to the 19th century):

- Linear equations: China, Borrel, Gosselin, Pell, ...
- Determinants: Maclaurin, Gauss, Cauchy, ...
- Eigenvalues and eigenvectors: Euler, Laplace, Cauchy
- Matrices: Gauss, Cayley, Jordan, ...
- Vectors: Newton, Argand, Hamilton
- Vector spaces: Grassmann, Dedekind, Steinitz, van der Waerden, ...

Summary of the course: lecture XV

Other ideas stemming from Euclid—non-Euclidean geometry and number theory (Antiquity to the 19th century):

- The Parallel Postulate: Gauss, Bolyai, Lobachevskii
- Elementary number theory: Euclid
- Perfect numbers: Euclid, Fermat, Mersenne, Euler
- Further number theory: Fermat, Euler, Gauss

Structure of the exam paper

Section A

- Six extracts given
- Choose two and comment on the context, content, and significance
- Each extract is worth 25 marks
- Each extract is typically one short paragraph — it will relate to a topic that we have studied, though you may not have seen the precise extract before
- By way of practice, choose any quotation or short extract that has appeared on the lecture slides

Section B

- Three essay topics given
- Choose one
- Answer worth 50 marks

Typical exam questions (Section B)

Q. Discuss, with reference to specific examples, how concept X (or terminology Y, or notation Z, ...) has developed between 1600 and 1900.

Q. Discuss with reference to specific examples, how attitudes towards X have changed between 1600 and 1900.

Q. Discuss the significance of text X.

Q. Describe some aspects of the work of major figure X.

Historiography of mathematics

According to the *OED*:

historiography, *n.*

1. The writing of history; written history.
2. The study of history-writing, esp. as an academic discipline.

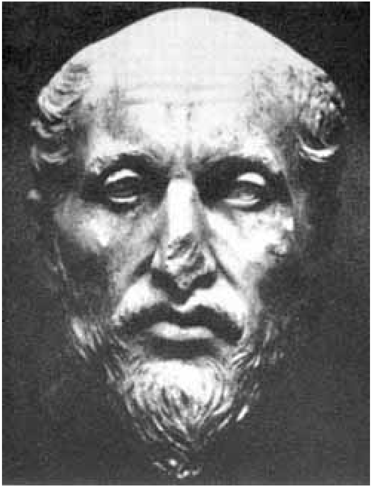
Ancient histories of mathematics

Aristotle (384–322 BC)

Eudemus (4th century BC)

- Student and editor of Aristotle
- *History of Arithmetic*
- *History of Geometry*
- *History of Astronomy*

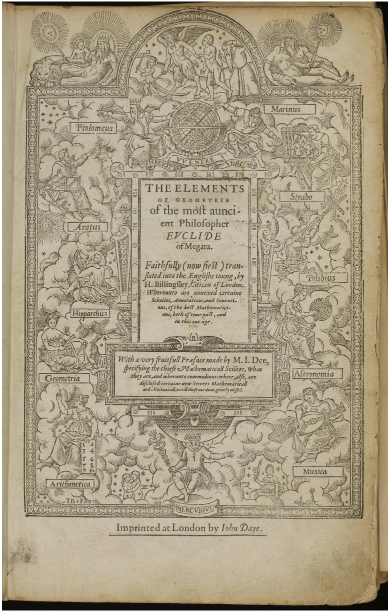
Biographical background



Proclus's commentary on Euclid's *Elements* (5th century AD)

- (Spurious?) biographical details
- Built on anecdotes provided by Pappus (4th century AD)

Later historical attributions



a full understanding of geometry "requireth diligent studie and reading of olde auncient authors"

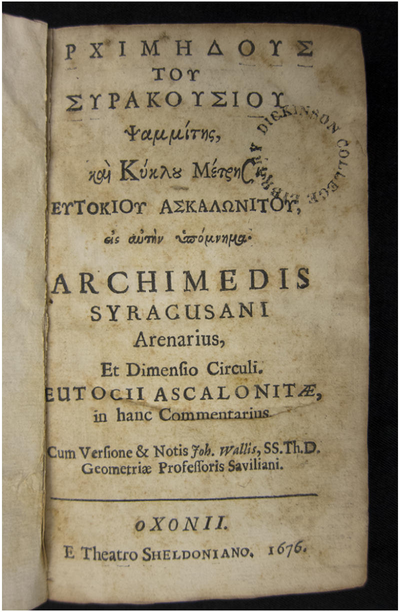
Renaissance humanist attitudes

Sir Henry Savile (1549–1622)

The teaching of mathematics should be founded on humanist principles:

- it should stem from the works of classical antiquity;
- scholars ought to have a concern for the history of their subject;
- they should actively seek to restore and edit surviving texts.

Renaissance humanist attitudes



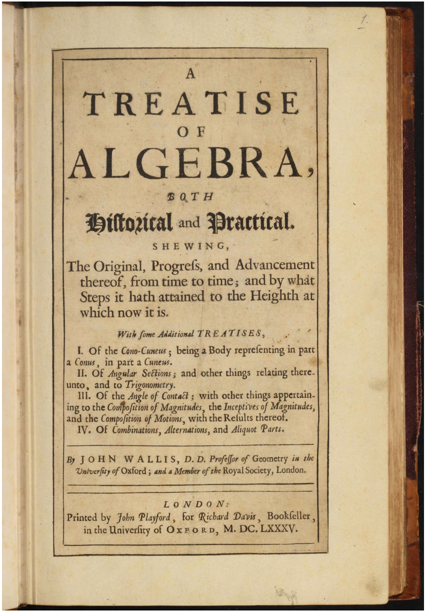
ΡΧΙΜΗΔΟΥΣ
ΤΟΥ
ΣΥΡΑΚΟΥΣΙΟΥ
Ψαμμίτης,
καὶ Κύκλου Μέτρησις,
ΕΥΤΟΚΙΟΥ ΑΣΚΑΛΩΝΙΤΟΥ,
εἰς αὐτὴν ὑπόμνημα.

ARCHIMEDIS
SYRACUSANI
Arenarius,
Et Dimenſio Circuli.
EUTOCII ASCALONITÆ,
in hanc Commentarius.

Cum Verſione & Notis *Joh. Wallis*, SS.Th.D.
Geometriæ Profeſſoris Saviliani.

OXONII.
E Theatro SHELDONIANO. 1676.

Nationalist attitudes



A

TREATISE

OF

ALGEBRA,

BOTH

Historical and Practical.

SHEWING,

The Original, Progreſs, and Advancement thereof, from time to time; and by what Steps it hath attained to the Heighth at which now it is.

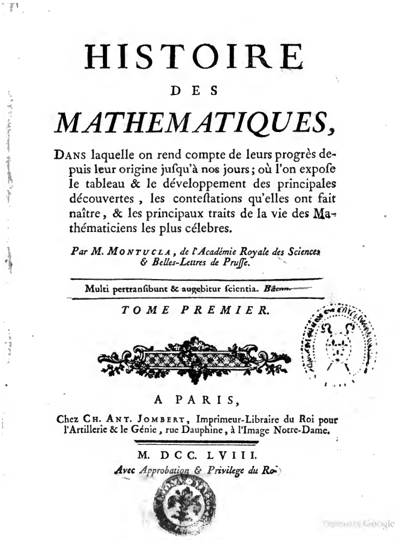
With ſome Additional TREATISES,

I. Of the *Cono-Cuneus*; being a Body repreſenting in part a *Conus*, in part a *Cuneus*.
II. Of *Angular Sections*; and other things relating thereunto, and to *Trigonometry*.
III. Of the *Angle of Contact*; with other things appertaining to the *Compoſition of Magnitudes*, the *Inceptives of Magnitudes*, and the *Compoſition of Motions*, with the Reſults thereof.
IV. Of *Combinations*, *Alternations*, and *Aliquot Parts*.

By JOHN WALLIS, *D. D. Profeſſor of* Geometry *in the Univerſity of* Oxford; *and a Member of the* Royal Society, London.

LONDON:
Printed by *John Playford*, for *Richard Davis*, Bookſeller, in the Univerſity of OXFORD, M. DC. LXXXV.

Comprehensive histories of mathematics



HISTOIRE

DES

MATHEMATIQUES,

DANS laquelle on rend compte de leurs progrès depuis leur origine jusqu'à nos jours; où l'on expose le tableau & le développement des principales découvertes, les contestations qu'elles ont fait naître, & les principaux traits de la vie des Mathématiciens les plus célebres.

Par M. MONTUCLA, de l'Académie Royale des Sciences & Belles-Lettres de Prusse.

Multi pertransibunt & augebitur scientia. *Bacon.*

TOME PREMIER.

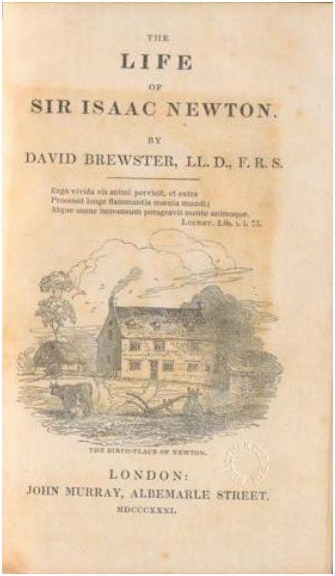
A PARIS,

Chez CH. ANT. JOMBERT, Imprimeur-Libraire du Roi pour l'Artillerie & le Génie, rue Dauphine, à l'Image Notre-Dame.

M. DCC. LVIII.

Avec Approbation & Privilege du Roi.

Lauding the great mathematicians



THE

LIFE

OF

SIR ISAAC NEWTON.

BY

DAVID BREWSTER, LL. D., F. R. S.

Ergo vivida vis animi pervicit, et extra
Processit longe flammantia mœnia mundi;
Atque omne immensum peragravit mente animoque.
Lucret. Lib. i. l. 73.

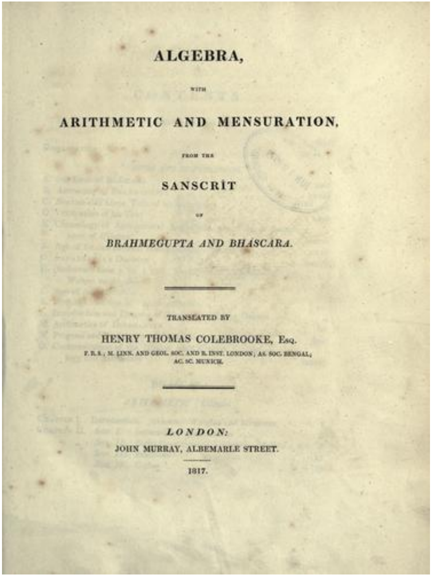
THE BIRTH-PLACE OF NEWTON.

LONDON:
JOHN MURRAY, ALBEMARLE STREET.
MDCCCXXXI.

Adding greater nuance

See: Adrian Rice, ‘Augustus De Morgan: historian of science’, *History of Science* 34 (1996), 201–240

Awareness of mathematics beyond Europe



ALGEBRA,

WITH

ARITHMETIC AND MENSURATION,

FROM THE

SANSCRIT

OF

BRAHMEGUPTA AND BHASCARA.

TRANSLATED BY

HENRY THOMAS COLEBROOKE, Esq.

F.R.S.; M. LINN. AND GEOL. SOC. AND R. INST. LONDON; AS. SOC. BENGAL; AC. SC. MUNICH.

LONDON:

JOHN MURRAY, ALBEMARLE STREET.

1817.

See: Ivahn Smadja, 'Sanskrit versus Greek 'proofs' : history of mathematics at the crossroads of philology and mathematics in nineteenth-century Germany', *Revue d'histoire des mathématiques* 21(2) (2015) 217–349

Anecdotal history

VORLESUNGEN

ÜBER

GESCHICHTE DER MATHEMATIK

VON

MORITZ CANTOR.

ERSTER BAND.

VON DEN ÄLTESTEN ZEITEN BIS ZUM JAHRE 1200 N. CHR.

LEIPZIG,

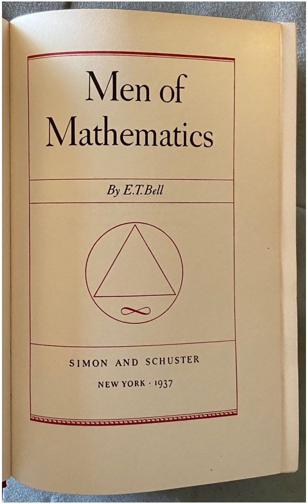
DRUCK UND VERLAG VON B. G. TEUBNER.

1880.

Professionalisation

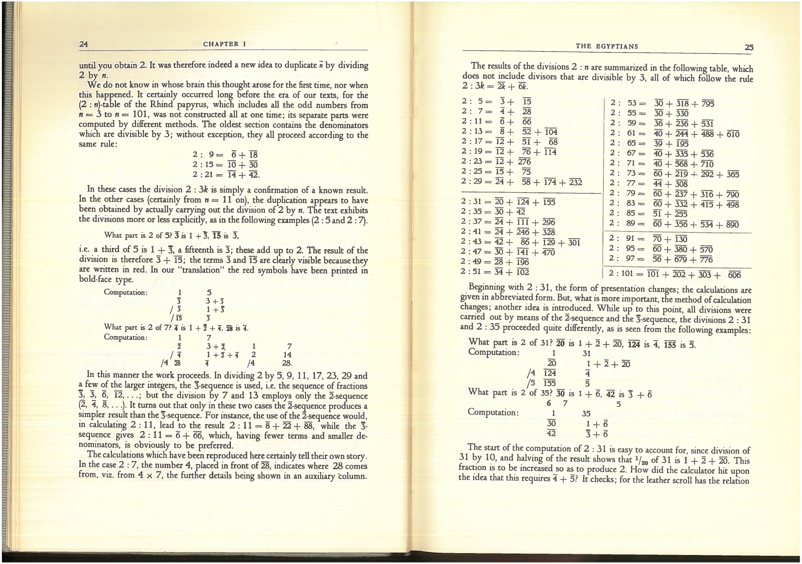
Otto Neugebauer (1899–1990)

Popularisation



- Humanisation of mathematics
- Mathematical myth-making
- Mathematical anecdotes for community-building
- Drawback: reinforcement of a particular view of the subject

Rewriting the history of mathematics



until you obtain 2. It was therefore indeed a new idea to duplicate $\overline{n}$ by dividing 2 by n.

We do not know in whose brain this thought arose for the first time, nor when this happened. It certainly occurred long before the era of our texts, for the $(2:n)$-table of the Rhind papyrus, which includes all the odd numbers from $n = 3$ to $n = 101$, was not constructed all at one time; its separate parts were computed by different methods. The oldest section contains the denominators which are divisible by 3; without exception, they all proceed according to the same rule:

$$2:9 = \overline{6} + \overline{18}$$
$$2:15 = \overline{10} + \overline{30}$$
$$2:21 = \overline{14} + \overline{42}.$$

In these cases the division $2:3k$ is simply a confirmation of a known result. In the other cases (certainly from $n = 11$ on), the duplication appears to have been obtained by actually carrying out the division of 2 by n. The text exhibits the divisions more or less explicitly, as in the following examples ($2:5$ and $2:7$).

What part is 2 of 5? $\mathbf{\overline{3}}$ is $1 + \overline{\overline{3}}$, $\mathbf{\overline{15}}$ is $\overline{3}$,

i.e. a third of 5 is $1 + \overline{\overline{3}}$, a fifteenth is $\overline{3}$; these add up to 2. The result of the division is therefore $\overline{3} + \overline{15}$; the terms $\overline{3}$ and $\overline{15}$ are clearly visible because they are written in red. In our "translation" the red symbols have been printed in bold-face type.

Computation:		
	1	5
	$\overline{\overline{3}}$	$3 + \overline{3}$
/	$\overline{3}$	$1 + \overline{\overline{3}}$
/	$\overline{15}$	$\overline{3}$

What part is 2 of 7? $\mathbf{\overline{4}}$ is $1 + \overline{\overline{2}} + \overline{4}$, $\mathbf{\overline{28}}$ is $\overline{4}$.

Computation:				
	1	7		
	$\overline{2}$	$3 + \overline{2}$	1	7
/	$\overline{4}$	$1 + \overline{2} + \overline{4}$	2	14
/4	$\overline{28}$	$\overline{4}$	/4	28.

In this manner the work proceeds. In dividing 2 by 5, 9, 11, 17, 23, 29 and a few of the larger integers, the $\overline{\overline{3}}$-sequence is used, i.e. the sequence of fractions $\overline{\overline{3}}, \overline{3}, \overline{6}, \overline{12}, \ldots$; but the division by 7 and 13 employs only the $\overline{2}$-sequence $(\overline{2}, \overline{4}, \overline{8}, \ldots)$. It turns out that only in these two cases the $\overline{2}$-sequence produces a simpler result than the $\overline{\overline{3}}$-sequence. For instance, the use of the $\overline{2}$-sequence would, in calculating $2:11$, lead to the result $2:11 = \overline{8} + \overline{22} + \overline{88}$, while the $\overline{\overline{3}}$-sequence gives $2:11 = \overline{6} + \overline{66}$, which, having fewer terms and smaller denominators, is obviously to be preferred.

The calculations which have been reproduced here certainly tell their own story. In the case $2:7$, the number 4, placed in front of $\overline{28}$, indicates where 28 comes from, viz. from 4×7, the further details being shown in an auxiliary column.

The results of the divisions $2:n$ are summarized in the following table, which does not include divisors that are divisible by 3, all of which follow the rule $2:3k = \overline{2k} + \overline{6k}$.

$2:5 = \overline{3} + \overline{15}$	$2:53 = \overline{30} + \overline{318} + \overline{795}$
$2:7 = \overline{4} + \overline{28}$	$2:55 = \overline{30} + \overline{330}$
$2:11 = \overline{6} + \overline{66}$	$2:59 = \overline{36} + \overline{236} + \overline{531}$
$2:13 = \overline{8} + \overline{52} + \overline{104}$	$2:61 = \overline{40} + \overline{244} + \overline{488} + \overline{610}$
$2:17 = \overline{12} + \overline{51} + \overline{68}$	$2:65 = \overline{39} + \overline{195}$
$2:19 = \overline{12} + \overline{76} + \overline{114}$	$2:67 = \overline{40} + \overline{335} + \overline{536}$
$2:23 = \overline{12} + \overline{276}$	$2:71 = \overline{40} + \overline{568} + \overline{710}$
$2:25 = \overline{15} + \overline{75}$	$2:73 = \overline{60} + \overline{219} + \overline{292} + \overline{365}$
$2:29 = \overline{24} + \overline{58} + \overline{174} + \overline{232}$	$2:77 = \overline{44} + \overline{308}$
	$2:79 = \overline{60} + \overline{237} + \overline{316} + \overline{790}$
$2:31 = \overline{20} + \overline{124} + \overline{155}$	$2:83 = \overline{60} + \overline{332} + \overline{415} + \overline{498}$
$2:35 = \overline{30} + \overline{42}$	$2:85 = \overline{51} + \overline{255}$
$2:37 = \overline{24} + \overline{111} + \overline{296}$	$2:89 = \overline{60} + \overline{356} + \overline{534} + \overline{890}$
$2:41 = \overline{24} + \overline{246} + \overline{328}$	
$2:43 = \overline{42} + \overline{86} + \overline{129} + \overline{301}$	$2:91 = \overline{70} + \overline{130}$
$2:47 = \overline{30} + \overline{141} + \overline{470}$	$2:95 = \overline{60} + \overline{380} + \overline{570}$
$2:49 = \overline{28} + \overline{196}$	$2:97 = \overline{56} + \overline{679} + \overline{776}$
$2:51 = \overline{34} + \overline{102}$	$2:101 = \overline{101} + \overline{202} + \overline{303} + \overline{606}$

Beginning with $2:31$, the form of presentation changes; the calculations are given in abbreviated form. But, what is more important, the method of calculation changes; another idea is introduced. While up to this point, all divisions were carried out by means of the $\overline{2}$-sequence and the $\overline{\overline{3}}$-sequence, the divisions $2:31$ and $2:35$ proceeded quite differently, as is seen from the following examples:

What part is 2 of 31? $\mathbf{\overline{20}}$ is $1 + \overline{2} + \overline{20}$, $\mathbf{\overline{124}}$ is $\overline{4}$, $\mathbf{\overline{155}}$ is $\overline{5}$.

Computation:		
	1	31
	$\overline{20}$	$1 + \overline{2} + \overline{20}$
/4	$\overline{124}$	$\overline{4}$
/5	$\overline{155}$	$\overline{5}$

What part is 2 of 35? $\mathbf{\overline{30}}$ is $1 + \overline{6}$, $\mathbf{\overline{42}}$ is $\overline{\overline{3}} + \overline{6}$

6 7 5

Computation:		
	1	35
	$\overline{30}$	$1 + \overline{6}$
	$\overline{42}$	$\overline{\overline{3}} + \overline{6}$

The start of the computation of $2:31$ is easy to account for, since division of 31 by 10, and halving of the result shows that $^1/_{20}$ of 31 is $1 + \overline{2} + \overline{20}$. This fraction is to be increased so as to produce 2. How did the calculator hit upon the idea that this requires $\overline{4} + \overline{5}$? It checks; for the leather scroll has the relation

Rewriting the history of mathematics

On the Need to Rewrite the History of Greek Mathematics

Sabetai Unguru

Communicated by W. Hartner

'History is the most fundamental science, for there is no human knowledge which cannot lose its scientific character when men forget the conditions under which it originated, the questions which it answered, and the function it was created to serve. A great part of the mysticism and superstition of educated men consists of knowledge which has broken loose from its historical moorings.'

Benjamin Farrington[1]

'It would not occur to the modern mathematician, who uses algebraic symbols, that one type of geometrical progression [*i.e.*, 1, 2, 4, 8] could be more perfect or better deserving of the name than another. For this reason algebraic symbols should not be employed in interpreting such a passage as ours [*i.e.*, Plato, *Timaeus*, 32A, B].'

Francis M. Cornford[2]

'Any historian of mathematics conscious of the perils and pitfalls of Whig history quickly discovers that the translation of past mathematics into modern symbolism and terminology represents the greatest danger of all. The symbols and terms of modern mathematics are the bearers of its concepts and methods. Their application to historical material always involves the risk of imposing on that material, a content it does not in fact possess.'

Michael S. Mahoney[3]

The previous string of quotations is (most certainly) **not** illustrative of the ways in which the history of mathematics has traditionally been written. The authors of the quotations themselves have not always practiced what they occa-

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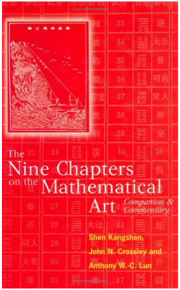
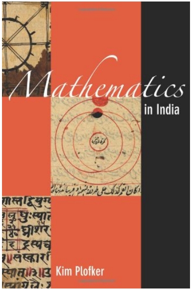
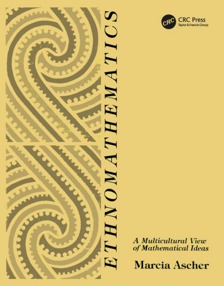
1 *Greek Science Its Meaning For Us* (Harmondsworth: Penguin Books, 1953), 311.
2 *Plato's Cosmology* (New York: The Liberal Arts Press, 1957), 49.
3 *The Mathematical Career of Pierre de Fermat (1601–1665)* (Princeton, N.J.: Princeton University Press, 1973), XII–XIII.

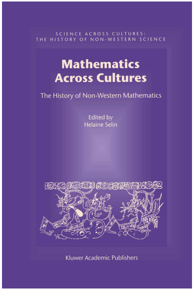
Sabetai Unguru, 'On the need to rewrite the history of Greek mathematics', *Archive for History of Exact Sciences* 15 (1975), 67–114

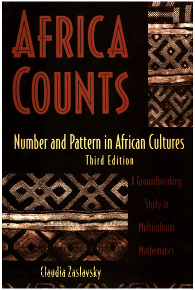
A broader perspective

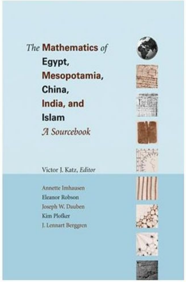




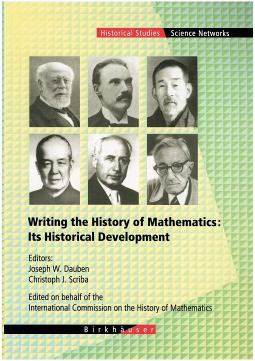


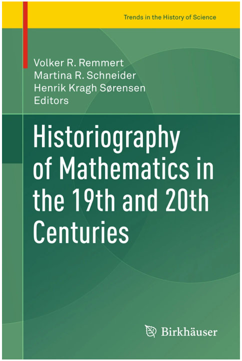


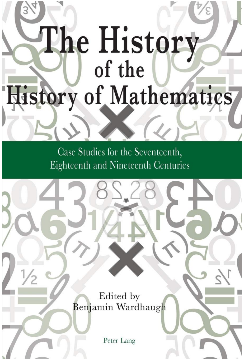




Historiography of mathematics: references







HT reading course: content

The quest for Fermat's Last Theorem

The main texts that we will read:

1. Euclid on Pythagorean triples;
2. Diophantus on problems of squares;
3. Pierre de Fermat's extension of ideas from Diophantus;
4. Leonhard Euler's attempts to fill the gaps in Fermat's work;
5. Sophie Germain's attempts to prove FLT;
6. E. E. Kummer's attempts to prove FLT.

As during the lecture course, the emphasis will be on the use of original sources — not only those mentioned above, but also any other relevant materials that may arise.

HT reading course: arrangements

Seminars: weekly classes on Friday mornings of an hour and a half each

(Note that these will timetabled with the lectures as two sessions per week, but you only need to attend one of these — sign up as you would for intercollegiate classes.)

Essays: up to 2,000 words to be submitted in advance for discussion in the seminars in weeks 3, 5 and 7

(Further details can be found on the course webpage.)

Assessment: extended essay (3,000 words), details of which will be announced on Monday of week 7. To be submitted by 12 noon on Monday of week 10.

HT reading course: vacation work

Details of vacation reading are on the course webpage

The British Society for the History of Mathematics:

www.bshm.ac.uk

BSHM undergraduate essay prize

http://www.bshm.ac.uk/undergraduate-essay-prize