Professor Arguz **B3.2 Geometry of Surfaces** **MT 2025**

Problem Sheet 2 Parts A and C

Part A.

1.

a) State the definition of the Euler characteristic of a surface.

b) Using the triangulation of a sphere which looks like the regular octahedron, compute the Euler characteristic of the sphere. How many vertices, edges, and faces are there?

c) Let M be a surface obtained as follows: Take the top half of a sphere which has a circle (the equator) as its boundary. Then, identify every point on the equator with its antipodal, the opposite point on the equator. Note that the resulting surface M does not have any boundary. Using the triangulation of the sphere given in part b), compute the Euler characteristic of M.

Part C.

1. Let X be the compact, connected surface obtained from a planar model given by a single polygon in the plane whose boundary word (read cyclically with orientations) is

$$abca^{-1}d^{-1}ec^{-1}b^{-1}e^{-1}d.$$

That is, the sides of the polygon are identified in pairs according to this word.

1. Compute the Euler characteristic $\chi(X)$.
2. Determine whether X is orientable.
3. Identify X up to homeomorphism in the classification of compact surfaces.