Professor Arguz **B3.2 Geometry of Surfaces** **MT 2025**

Problem Sheet 3 Parts A and C

Part A.

1.

Let S be the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, \qquad a, b, c > 0,$$

and let K denote its Gaussian curvature. Show that

$$\iint_S K \, dA = 4\pi.$$

Would the same result hold for

$$S' = \{(x, y, z) \in \mathbb{R}^3 : \ x^2 + y^2 + z^4 = 1\}\,?$$

2. Show that there is no point on a smooth surface with Gaussian curvature 30 and mean curvature 1.

Part C.

1.

A parametrization of the unbounded cylinder S of radius a is

$$\mathbf{r}(\theta, z) = (a\cos\theta, a\sin\theta, z), \quad \theta \in [0, 2\pi), \ z \in \mathbb{R}.$$

(i) Find the first and second fundamental forms I, II for S.

(ii) Find the principal curvatures of S at the origin, using the shape operator $S = I^{-1}II$ (this convention is slightly different from that used in class).

(iii) Find the Gaussian and mean curvatures at the origin.