Professor Arguz **B3.2 Geometry of Surfaces** **MT 2025**

Problem Sheet 4 Parts A and C

Part A.

1. For each statement below, prove if the statement is true. Otherwise, disprove by giving a counterexample.

(i) If two surfaces S_1 and S_2 have the same first fundamental forms, then their Gaussian curvatures are the same.

(ii) If S is a minimal surface with Gaussian curvature zero, then S must be part of a plane.

2.

Let C be the cylinder

$$C = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 = 1\},$$

and let

$$\gamma(t) = \begin{pmatrix} \cos t \\ \sin t \\ at \end{pmatrix}, \qquad a > 0,$$

be a helix on C. Compute the geodesic curvature of γ.

Part C.

1.

A surface of revolution is generated by revolving a plane curve

$$\gamma(u) = (r(u), z(u)), \qquad r(u) \geq 0,$$

about the z-axis. A standard parametrization of the surface is

$$\mathbf{r}(u, \theta) = \big(r(u)\cos\theta,\ r(u)\sin\theta,\ z(u)\big), \qquad \theta \in [0, 2\pi).$$

A parallel of the surface is a curve obtained by fixing $u = u_0$ and varying θ:

$$\theta \longmapsto \mathbf{r}(u_0, \theta) = \big(r(u_0)\cos\theta,\ r(u_0)\sin\theta,\ z(u_0)\big).$$

Show that if all parallels of a surface of revolution are S are geodesics, then S must be a circular cylinder.