Math C5.4, Networks
Week 1a

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"I think the next century will be the century of complexity."

Stephen Hawking

com.plex
[adj., v. kuh m-pleks, kom-pleks; n. kom-pleks]

1) composed of many interconnected parts; compound; composite: a complex highway system

2) characterized by a very complicated or involved arrangement of parts, units, etc.: complex machinery

3) so complicated or intricate as to be hard to understand or deal with: a complex problem

Source: Dictionary.com

Box 1.1

"Complex systems consist of a large number of interacting components. The interactions give rise to emergent hierarchical structures. The components of the system and properties at systems level typically change with time."

H.J. Jensen, in Encyclopedia of Complexity and Systems Science

Importance of metaphors, analogies and common languages

LATEST NEWS

COBBS LAB, ISC-CNR

All together now. Flocks of starlings fly over Rome's city center.

How bird flocks are like liquid helium

Tool box

Agent-based and numerical simulations

(Non-linear) Dynamical systems

Stochastic processes

Data mining and optimisation

Networks

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Agent-based and numerical simulations
(Non-linear) Dynamical systems
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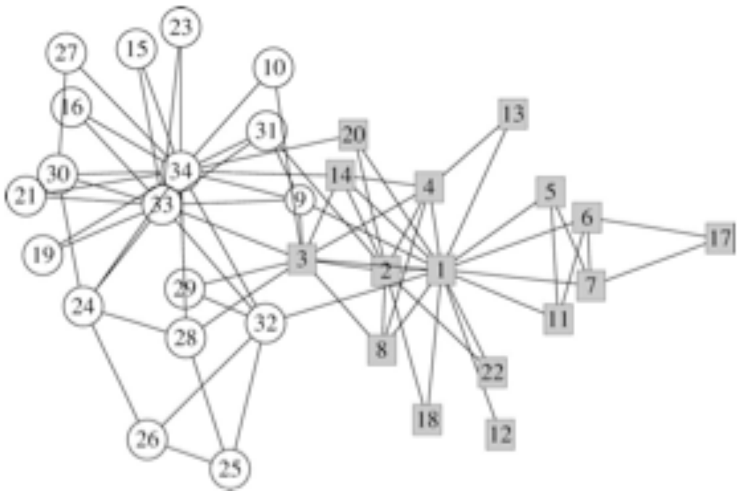
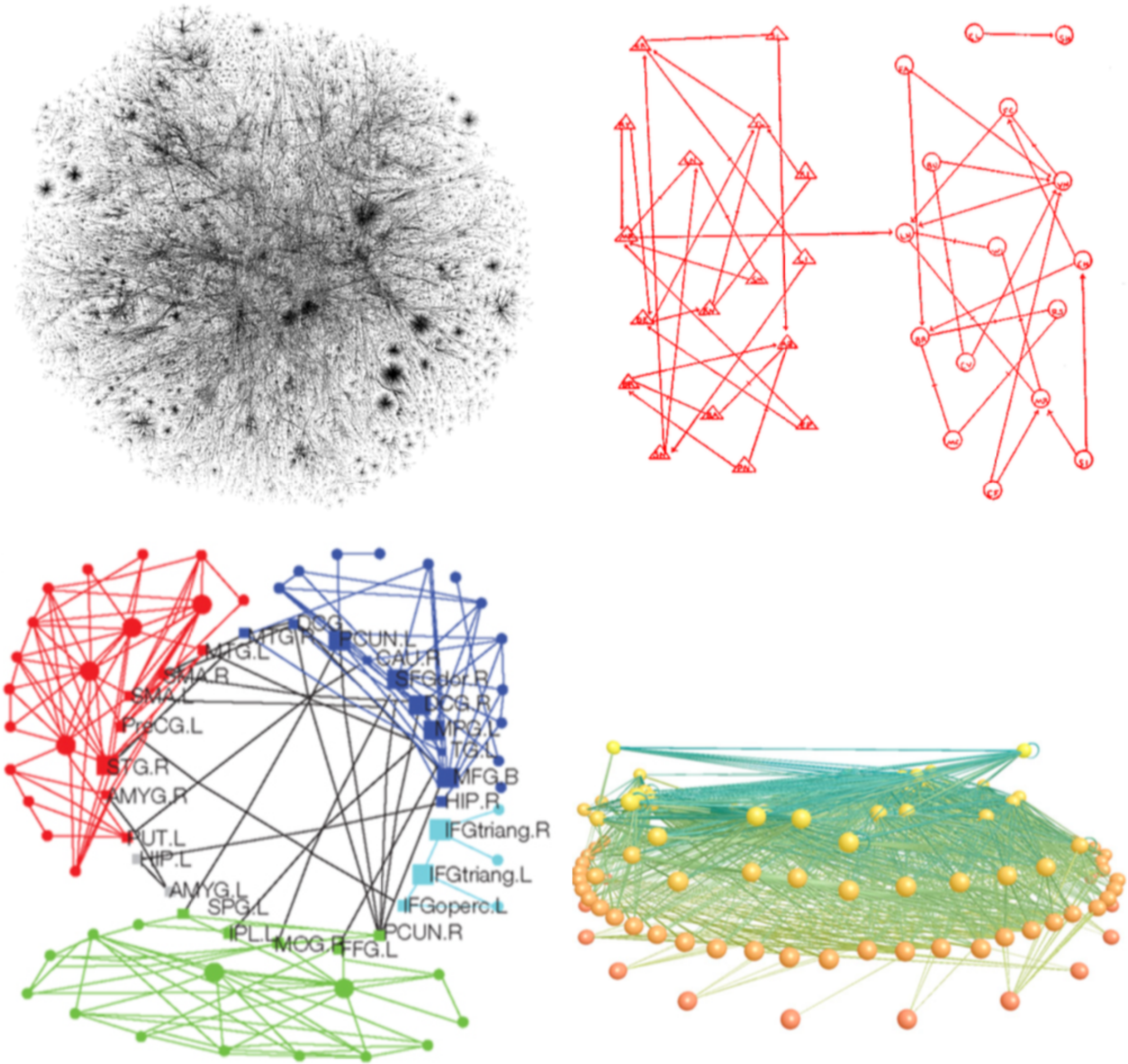
Networks



Social networks
- Collaboration networks
- Communication networks
- Online social networks

Biological networks
- Protein-protein interaction networks
- Neural networks
- Food webs


KONINGSBERGA



DCG
MTG.R
PCUN.L
MFG.L
CAU.R
SMA.R
SFGdor.R
SMA.L
DCG.R
PreCG.L
MPG.L
TG.L
STG.R
MFG.B
AMYG.R
HIP.R
IFGtriang.R
PUT.L
HIP.L
IFGtriang.L
AMYG.L
SPG.L
IFGoperc.L
IPL.L
PCUN.R
MOG.R
FFG.L

Relation between structure and (linear) dynamics

$$\mu \frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = L\mathbf{x}$$

Effect of structure on spreading:
SI, random walk, consensus, etc.

Uncover structure from dynamics:
Pagerank, Markov stability, etc.

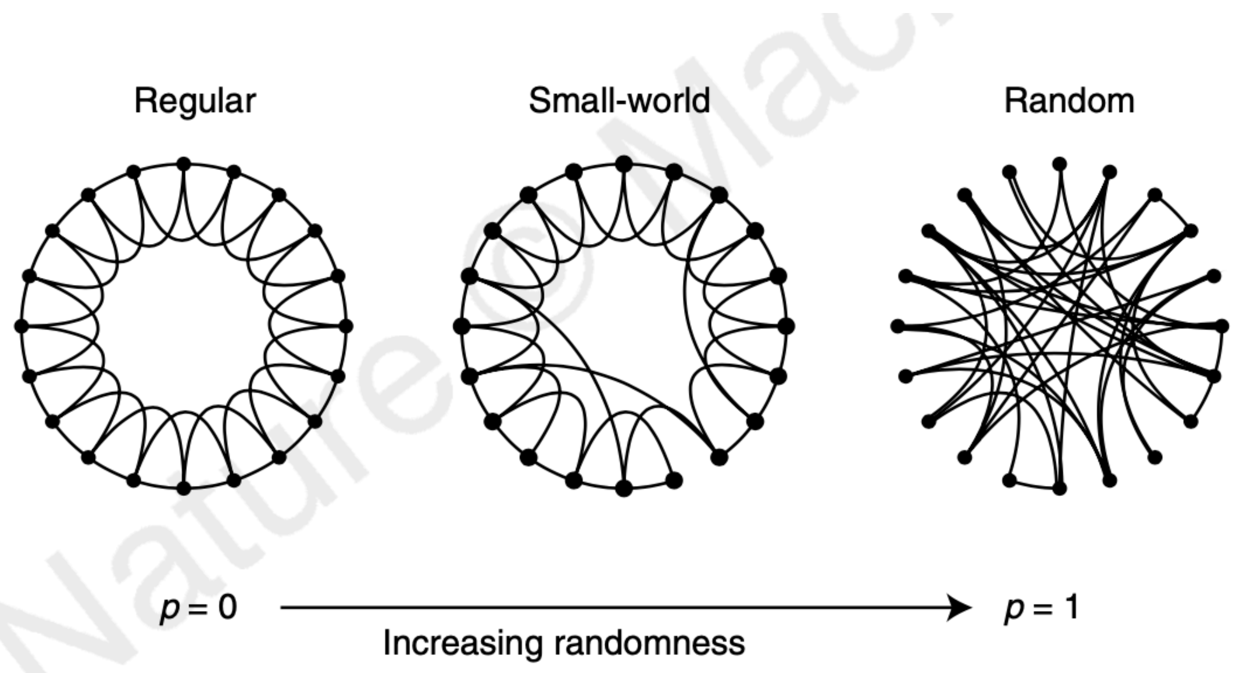
Real-world networks exhibit a balance between order and randomness

Published: 04 June 1998

Collective dynamics of 'small-world' networks

Duncan J. Watts & Steven H. Strogatz

Nature **393**, 440–442(1998) | Cite this article

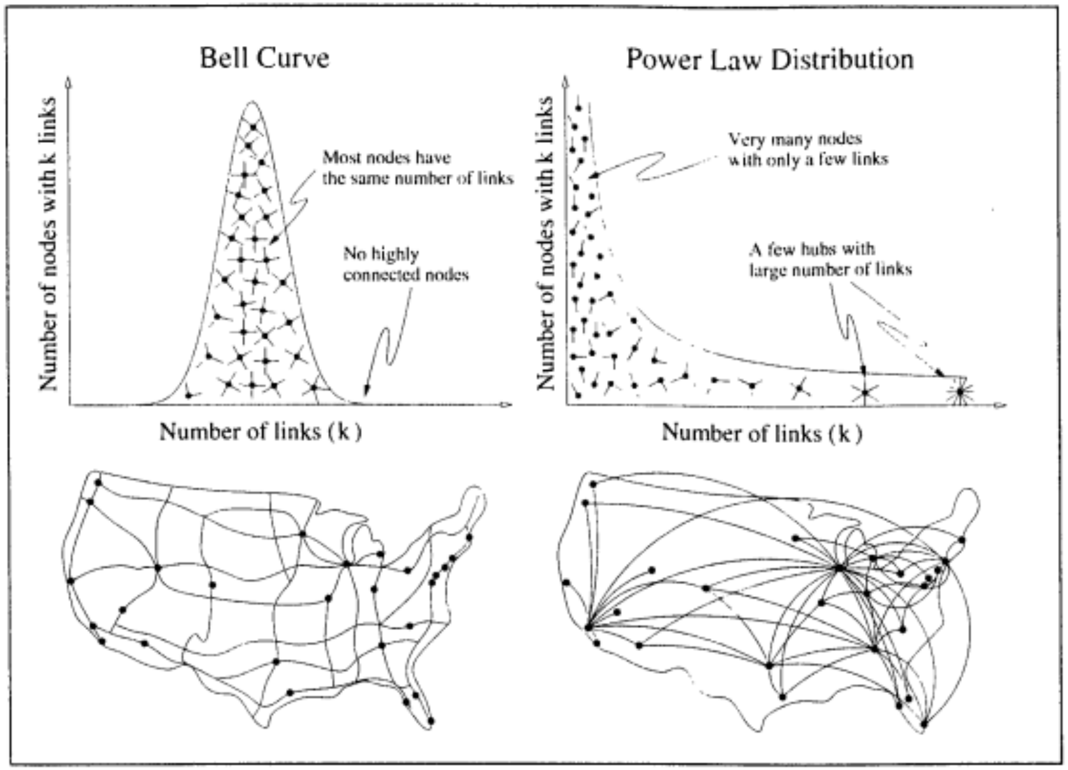


Week 1:

I: Introduction to the course

II: Power laws and basic statistical models

Clauset, Aaron, Cosma Rohilla Shalizi, and Mark EJ Newman. "Power-law distributions in empirical data." *SIAM review* 51.4 (2009): 661-703.



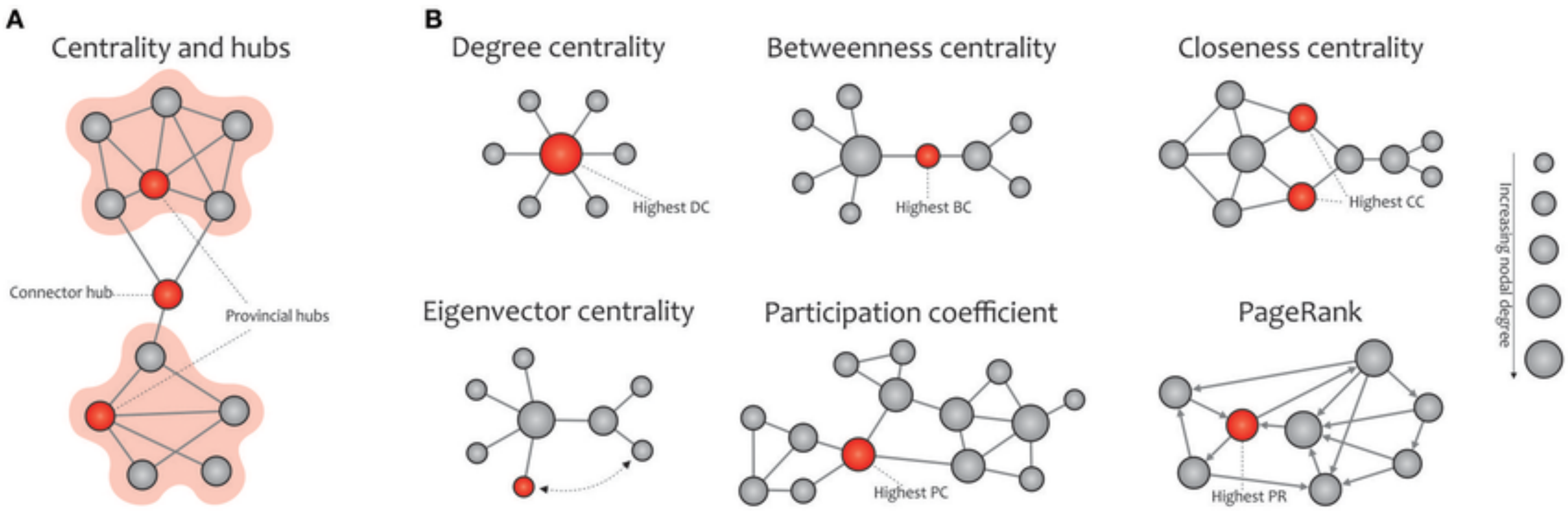
Week 2:

I: Basic network properties, walks and paths

Newman, Mark. Networks. Oxford university press, 2018. Chapter 6

II: Centrality and basic spectral properties

Newman, Mark. Networks. Oxford university press, 2018. Chapter 7



Week 3:

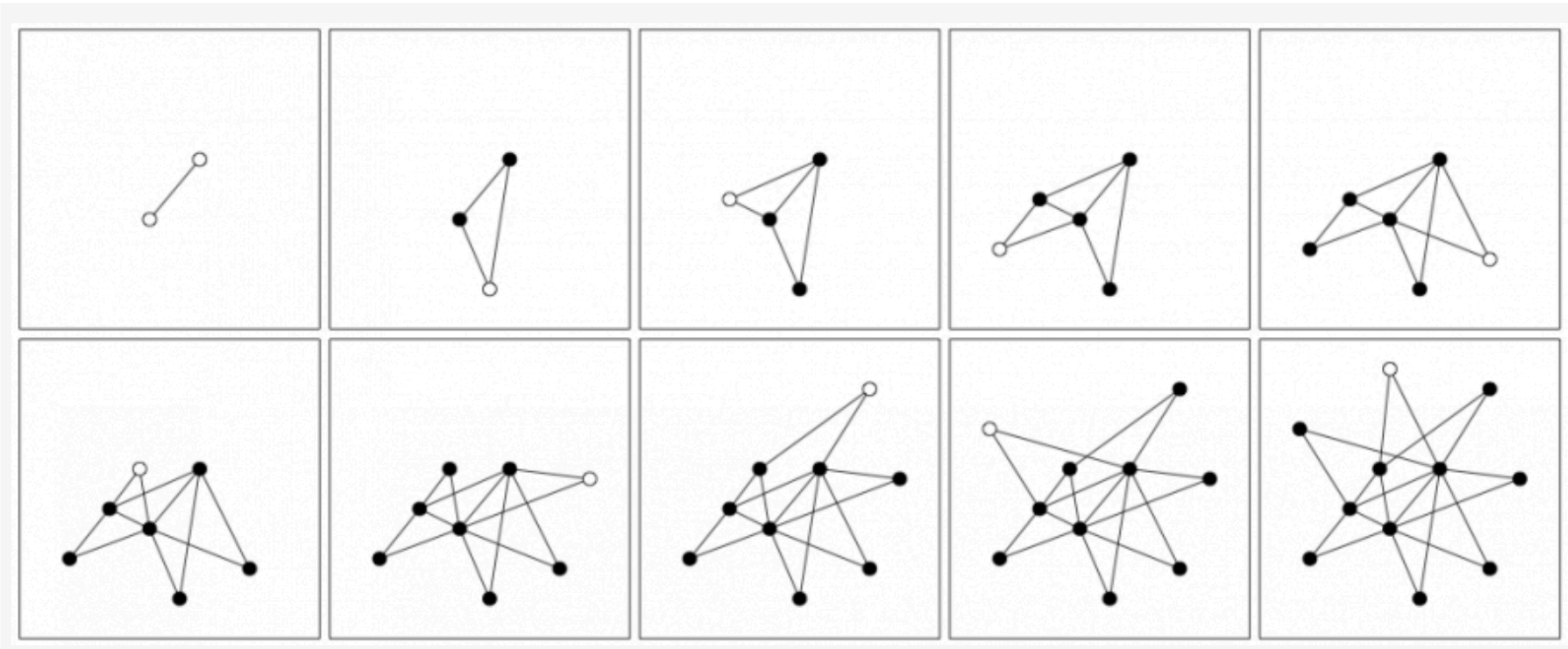
I: Random graph models

Newman, Mark. Networks. Oxford university press, 2018. Chapters 12 and 13

II: Mechanistic growing network models

Newman, Mark. Networks. Oxford university press, 2018. Chapter 14

Krapivsky, Pavel L., and Sidney Redner. "Network growth by copying." Physical Review E 71.3 (2005): 036118.



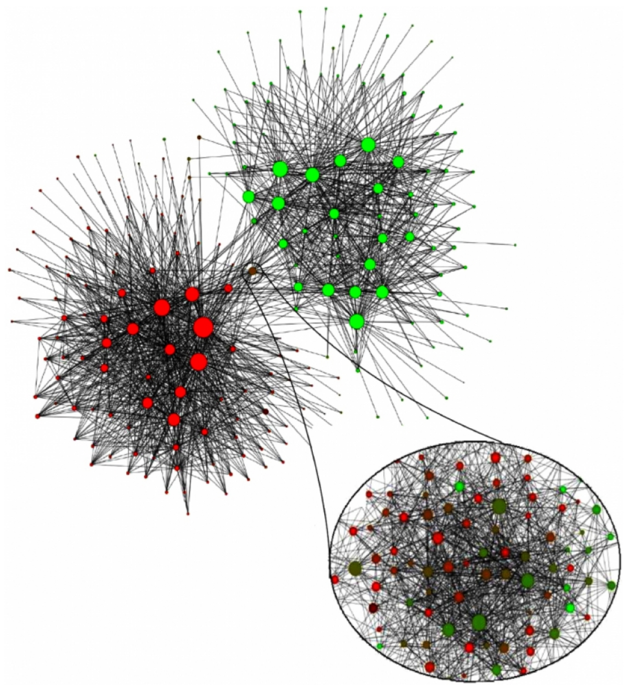
Week 4:

I: Community detection

Fortunato, Santo, and Darko Hric. "Community detection in networks: A user guide." *Physics reports* 659 (2016): 1-44.

II: Modularity optimisation

Lambiotte, Renaud, and Michael T. Schaub. Modularity and dynamics on complex networks. Cambridge University Press, 2021. Chapter 3



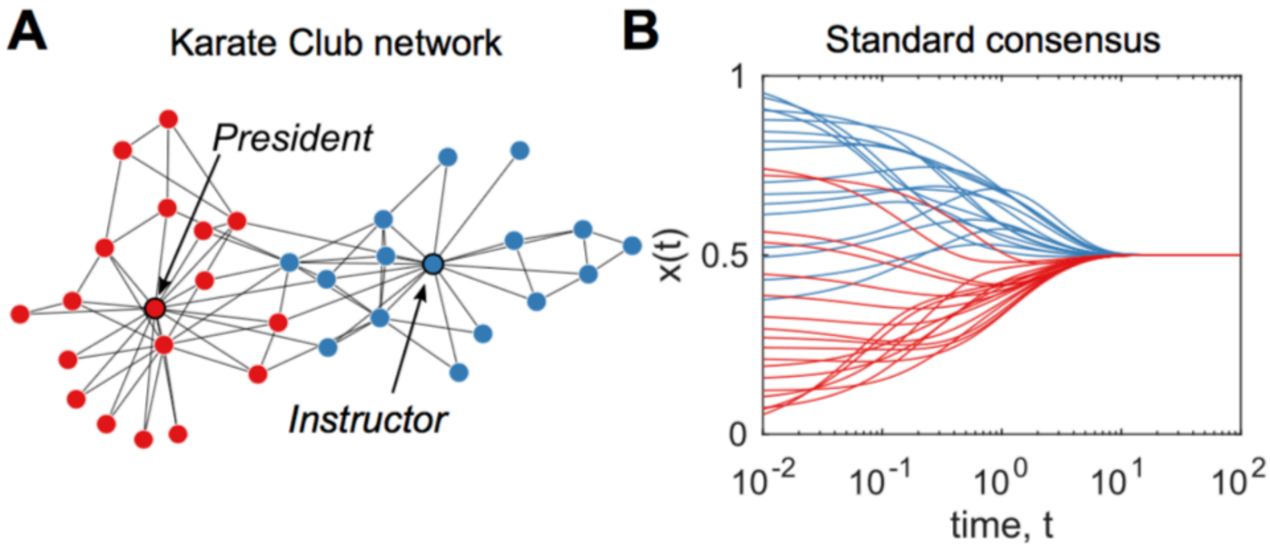
Week 5:

I: Consensus Dynamics and Random Walks

Lambiotte, Renaud, and Michael T. Schaub. Modularity and dynamics on complex networks. Cambridge University Press, 2021. Chapter 4

II: Time scale separation and structural communities

Lambiotte, Renaud, and Michael T. Schaub. Modularity and dynamics on complex networks. Cambridge University Press, 2021. Chapter 4



Week 6:

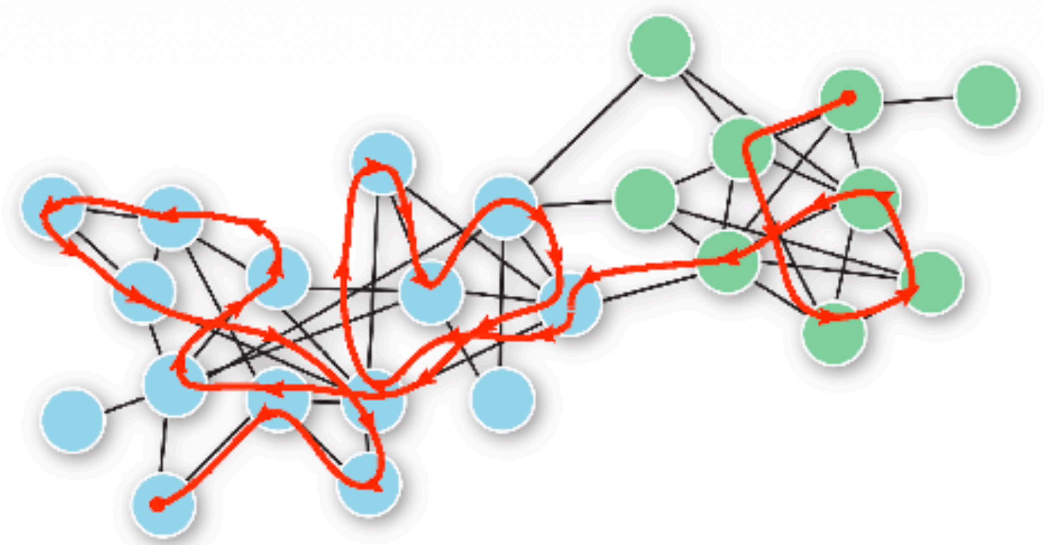
I: Dynamics for community detection, and the Map Equation

Masuda, Naoki, Mason A. Porter, and Renaud Lambiotte. "Random walks and diffusion on networks." *Physics reports* 716 (2017): 1-58.

Rosvall, Martin, Daniel Axelsson, and Carl T. Bergstrom. "The map equation." *The European Physical Journal Special Topics* 178.1 (2009): 13-23.

II: Markov stability

Lambiotte, Renaud, and Michael T. Schaub. Modularity and dynamics on complex networks. Cambridge University Press, 2021. Chapter 6



Week 7:

I: Non-linear dynamics and co-evolution

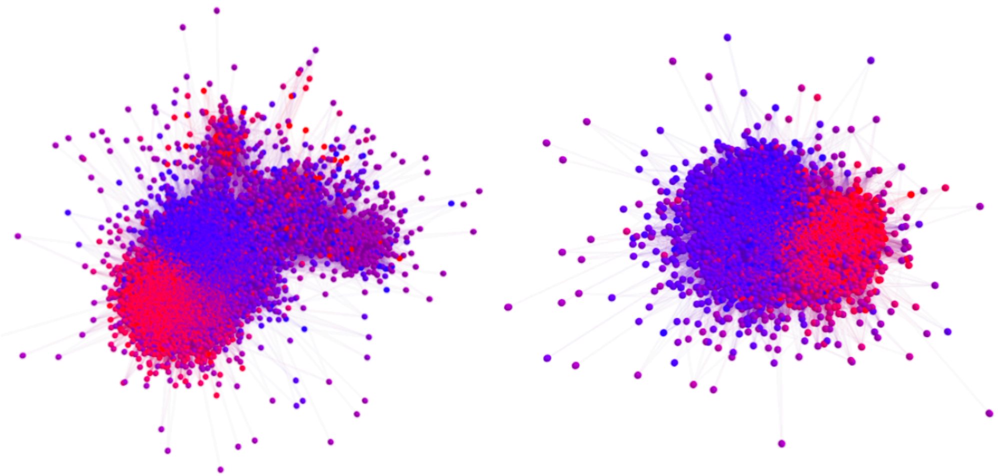
Porter, Mason A., and James P. Gleeson. "Dynamical systems on networks." *Frontiers in Applied Dynamical Systems: Reviews and Tutorials* 4 (2016): 29.

Gross, Thilo, and Bernd Blasius. "Adaptive coevolutionary networks: a review." Journal of the Royal Society Interface 5.20 (2008): 259-271.

II: Polarisation and network attributes

Peel, Leto, Jean-Charles Delvenne, and Renaud Lambiotte. "Multiscale mixing patterns in networks." *Proceedings of the National Academy of Sciences* 115.16 (2018): 4057-4062.

Baumann, Fabian, et al. "Modeling echo chambers and polarization dynamics in social networks." *Physical review letters* 124.4 (2020): 048301.



(a) Obamacare **(b)** Gun control

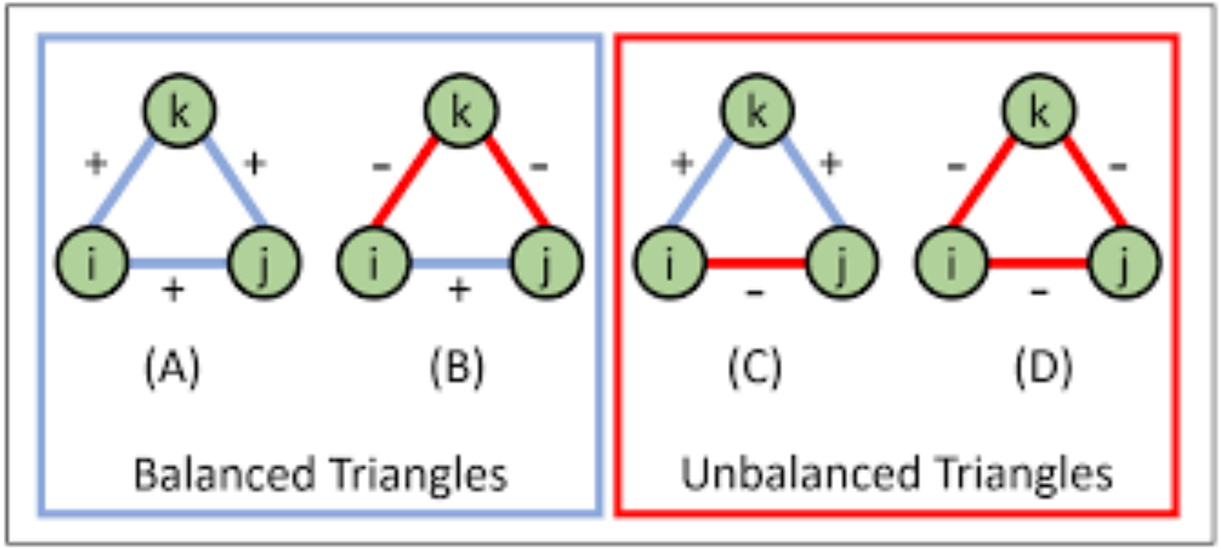
Week 8:

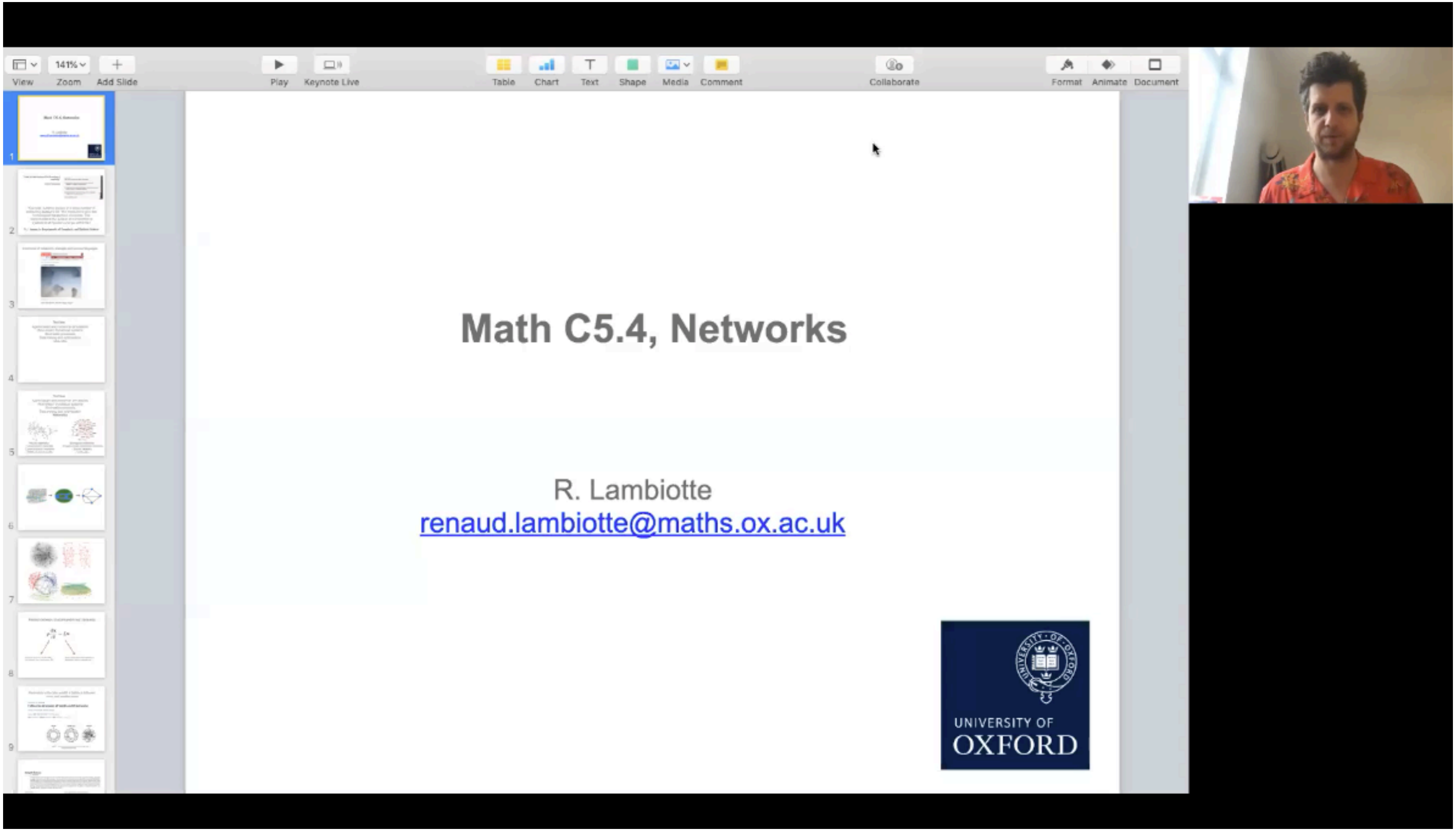
I: Modelling signed networks

Diaz-Diaz, Fernando, et al. "Signed Networks: theory, methods, and applications." *arXiv preprint arXiv:2511.17247* (2025).

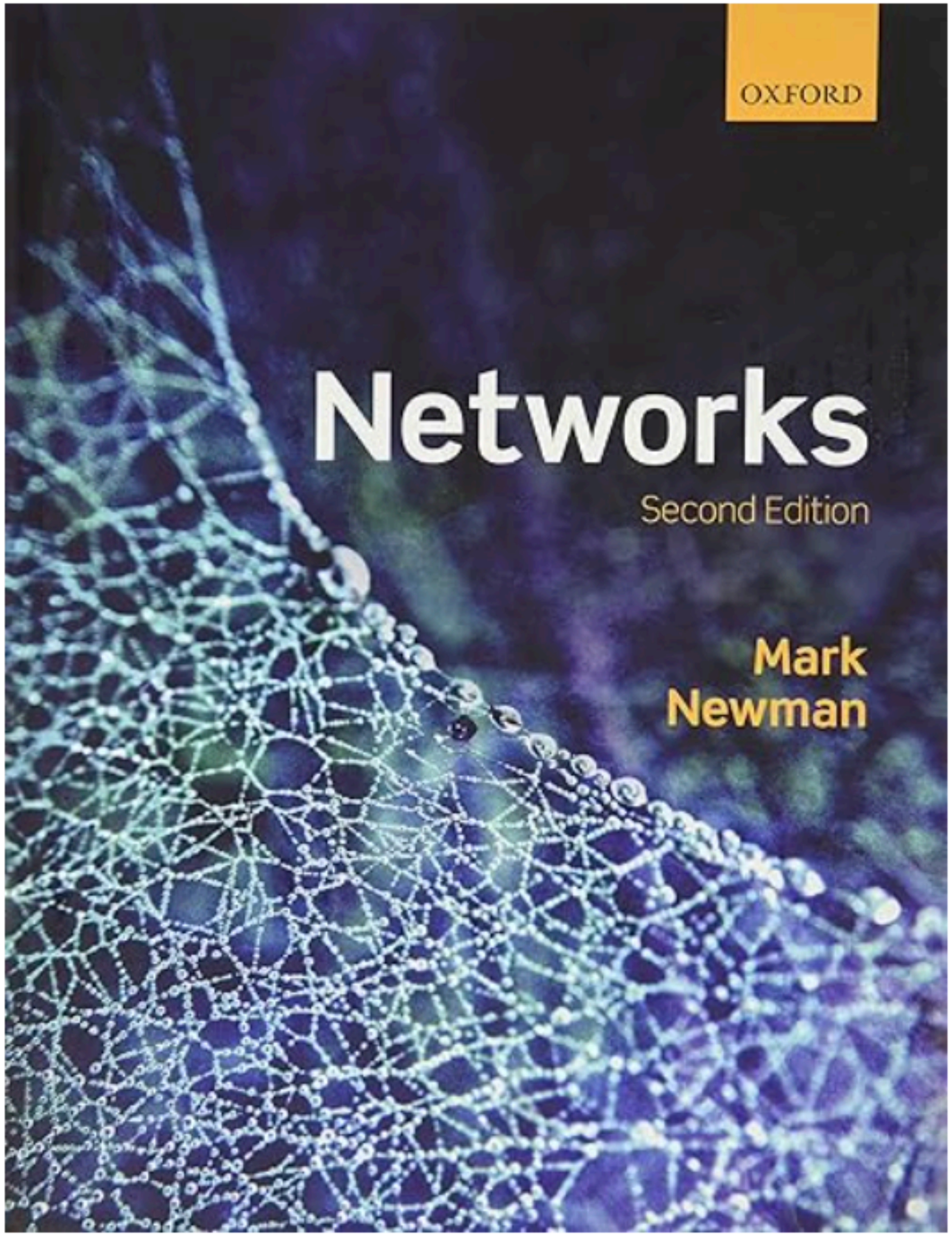
Tian, Yu, and Renaud Lambiotte. "Spreading and structural balance on signed networks." *SIAM Journal on Applied Dynamical Systems* 23.1 (2024): 50-80.

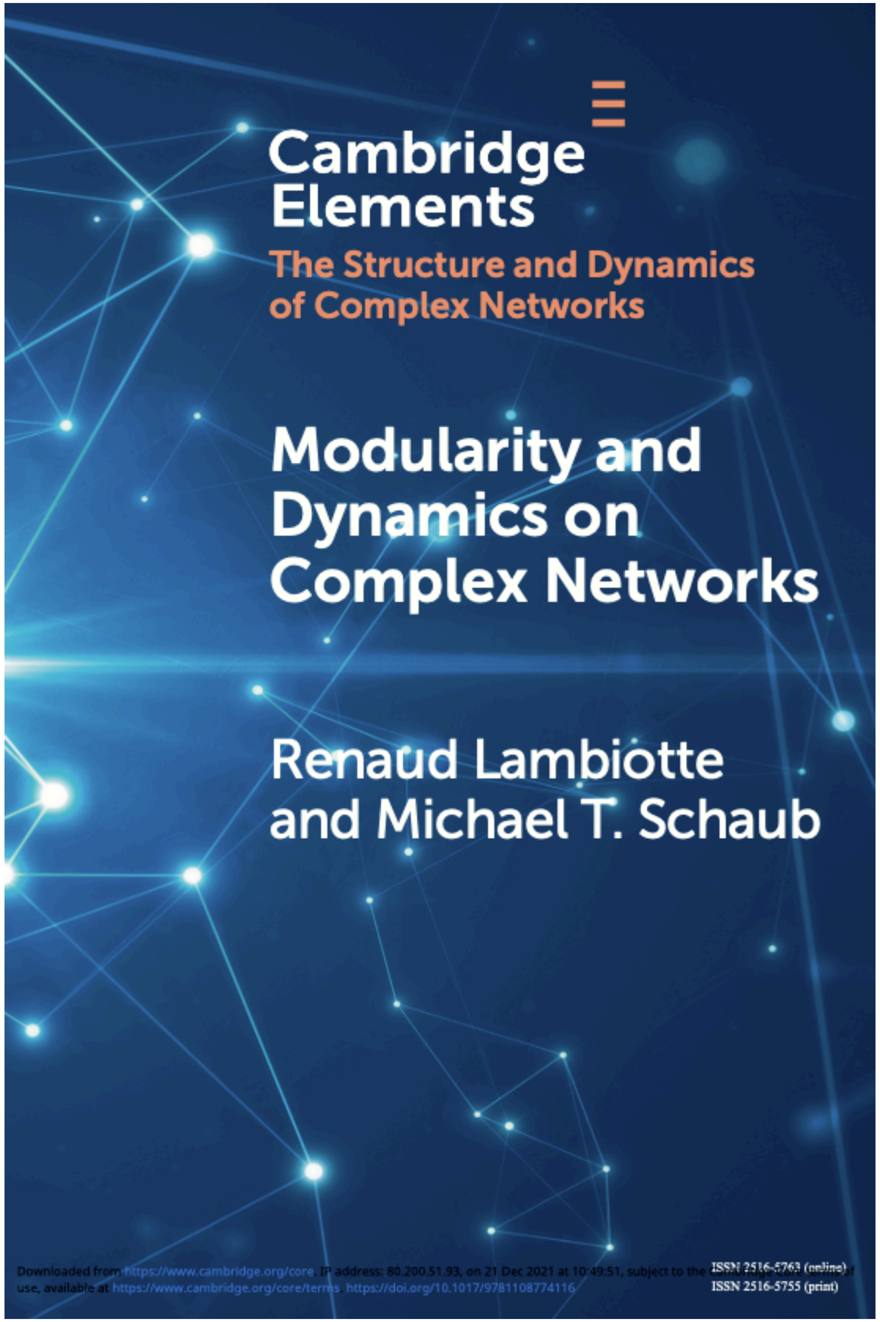
II: Conclusion and perspectives





https://www.youtube.com/watch?v=TQKgB0RnjeY&t=41s


OXFORD
Networks
Second Edition
Mark
Newman


Cambridge
Elements
The Structure and Dynamics
of Complex Networks
Modularity and
Dynamics on
Complex Networks
Renaud Lambiotte
and Michael T. Schaub
Downloaded from https://www.cambridge.org/core. IP address: 80.200.51.93, on 21 Dec 2021 at 10:49:51, subject to the
use, available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781108774116
ISSN 2516-5763 (online)
ISSN 2516-5755 (print)

Objectives

- Quantify the structural properties of nodes, for instance to estimate their importance or their centrality.

- Quantify the structure of networks, in particular their connectivity patterns and their statistical properties.

- Build random models of networks, in order to test the importance of structural factors and develop statistical tests.

- Extract groups of similar nodes, such as social circles, that is cluster the nodes into communities or modules.

- Identify the structural properties that affect the function and dynamics of the system, for instance the spreading of information or of a virus on the network.

Combination of numerics and analytics

3. *Connectedness.*
   (a) Draw a graph that is weakly connected but not strongly connected. Write down the adjacency matrix of this graph. What can happen to a random walk in such a graph, and what implication does this have for the asymptotic density of walkers on the nodes?
   (b) Consider the adjacency-matrix representation of a graph. What is the difference between the spectrum of directed networks versus that of undirected networks? (Recall that the set of eigenvalues of a matrix is called the *spectrum* of that matrix.)

4. *Clustering coefficients.* Draw a very small network in which the global clustering coefficient and mean local clustering coefficient have different values. Write down the adjacency matrix for this network.

5. *Small-world.* Ex.IV.4 : Generate graphs of the model c from Figure (12) and calculate the dependence of the network diameter and clustering coefficient on the number of shortcuts. Is this model, very similar to the so-called Watts-Strogatz model, a good model for a social network? Why or why not?

Examination

Mini-project!

Final Honour School of Mathematics Part C

C5.4 Networks

Non-backtracking random walks have, in recent years, attracted much attention in different aspects of network science, from community detection to the design of centrality measures and the study of epidemic processes.

Write a report on a specific subtopic within the general heading of non-backtracking random walks on networks. Your report must include some numerical simulations (which you produce) and must include a discussion on modelling issues, random-graph ensembles, and empirical data.

Your report should be in the format and style of an article for the journal Proceedings of the National Academy of Sciences, and the main text must be no more than 6 typeset pages and must use their LaTeX style files (a template and style files will be provided). The report must include all sections (abstract, significance statement, etc.) in papers published in that journal (2017 format of papers). It is permissible to include a section of Supplemental Information that shows additional figures and calculations. In your report, indicate explicitly which ideas are new and which come from existing sources, and use appropriate and explicit attributions for all references (which must include papers reporting original research) or anything else (e.g., including code and figures) from other sources.