

Mathematical modelling of cranial neural crest cell migration

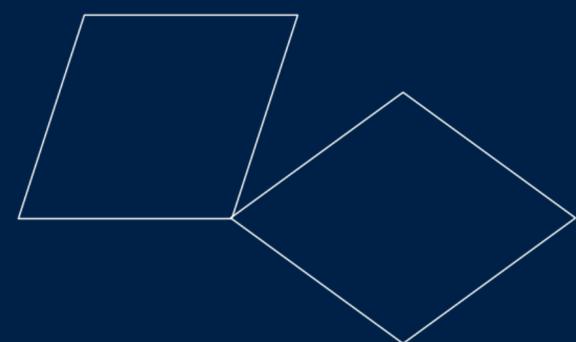


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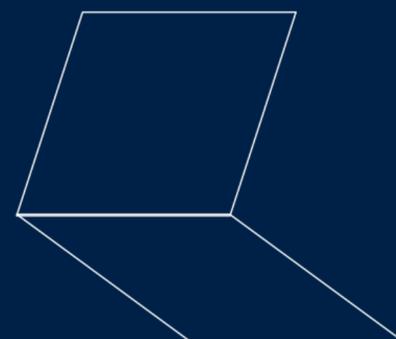
Samuel Johnson¹

MMSC Mathematical Modelling Case Study Project

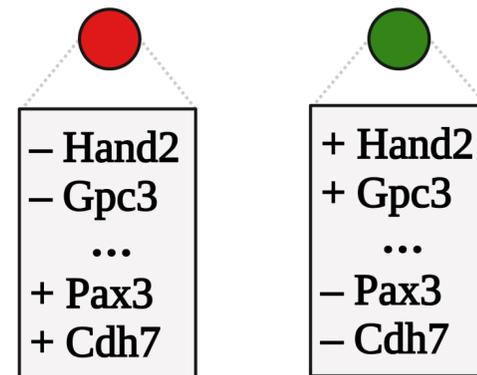
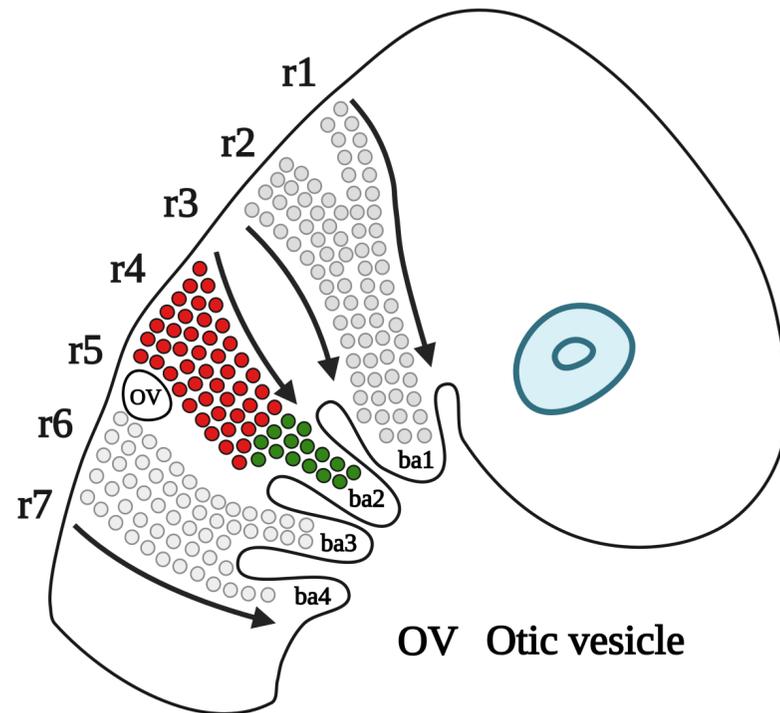
¹*WCMB, Mathematical Institute, University of Oxford*



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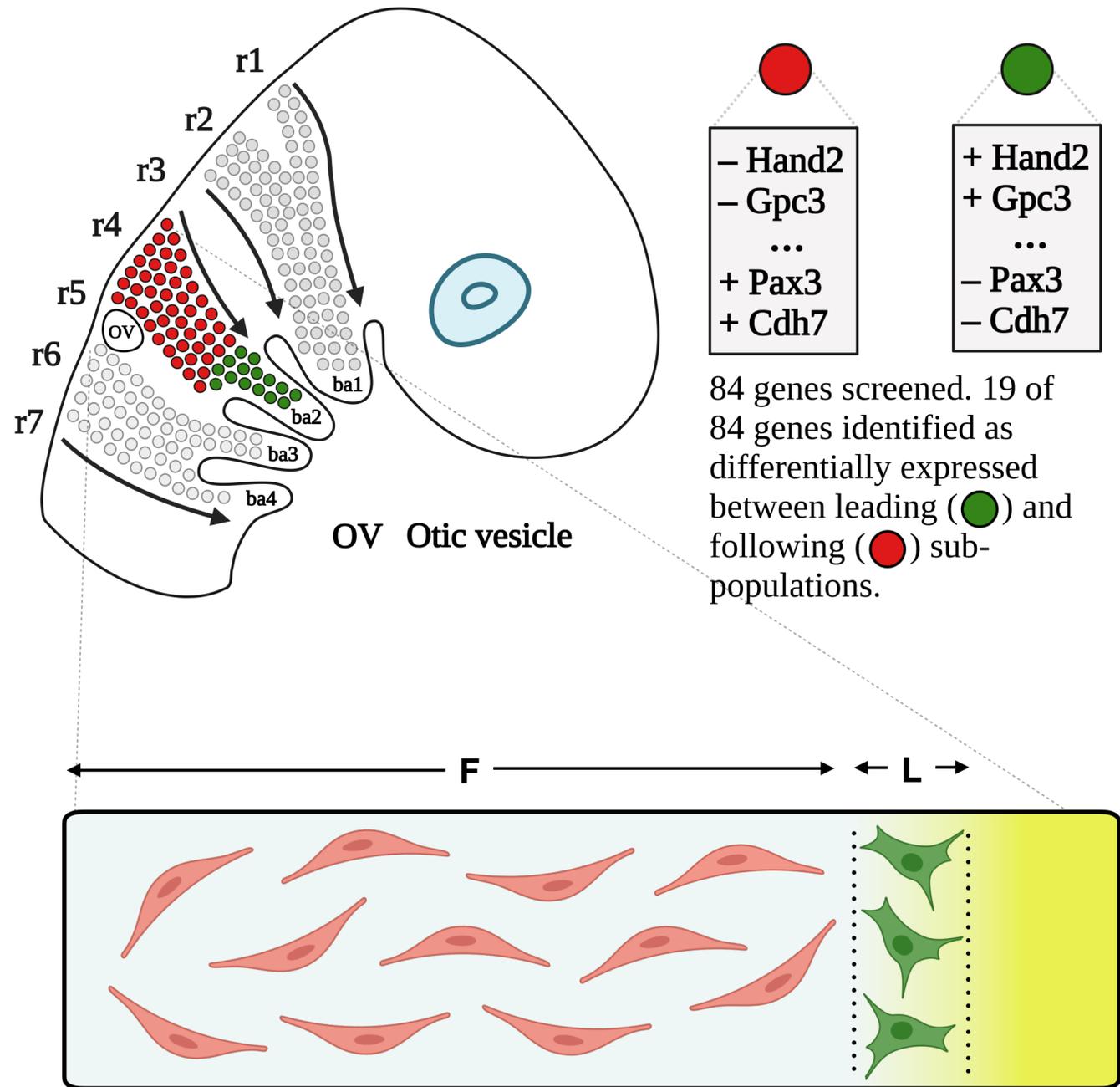
Collective cell migration in the Cranial Neural Crest



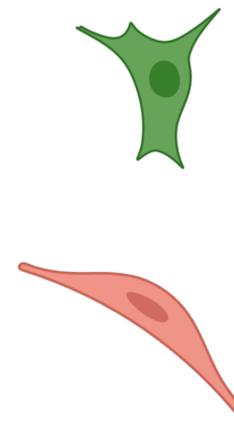
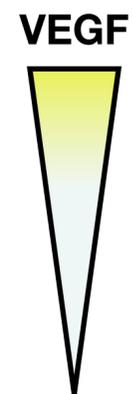
84 genes screened. 19 of 84 genes identified as differentially expressed between leading (●) and following (●) sub-populations.

- The cranial neural crest is a **highly migratory cell population** in **vertebrate embryogenesis** [1].
- Cranial neural crest cells **migrate collectively** towards targets in **branchial arches 1-4**.
- A better understanding of the mechanisms governing cranial neural crest cell migration may provide opportunities for the **development of novel therapeutics** to correct or prevent **aberrant cranial neural crest cell migration**.

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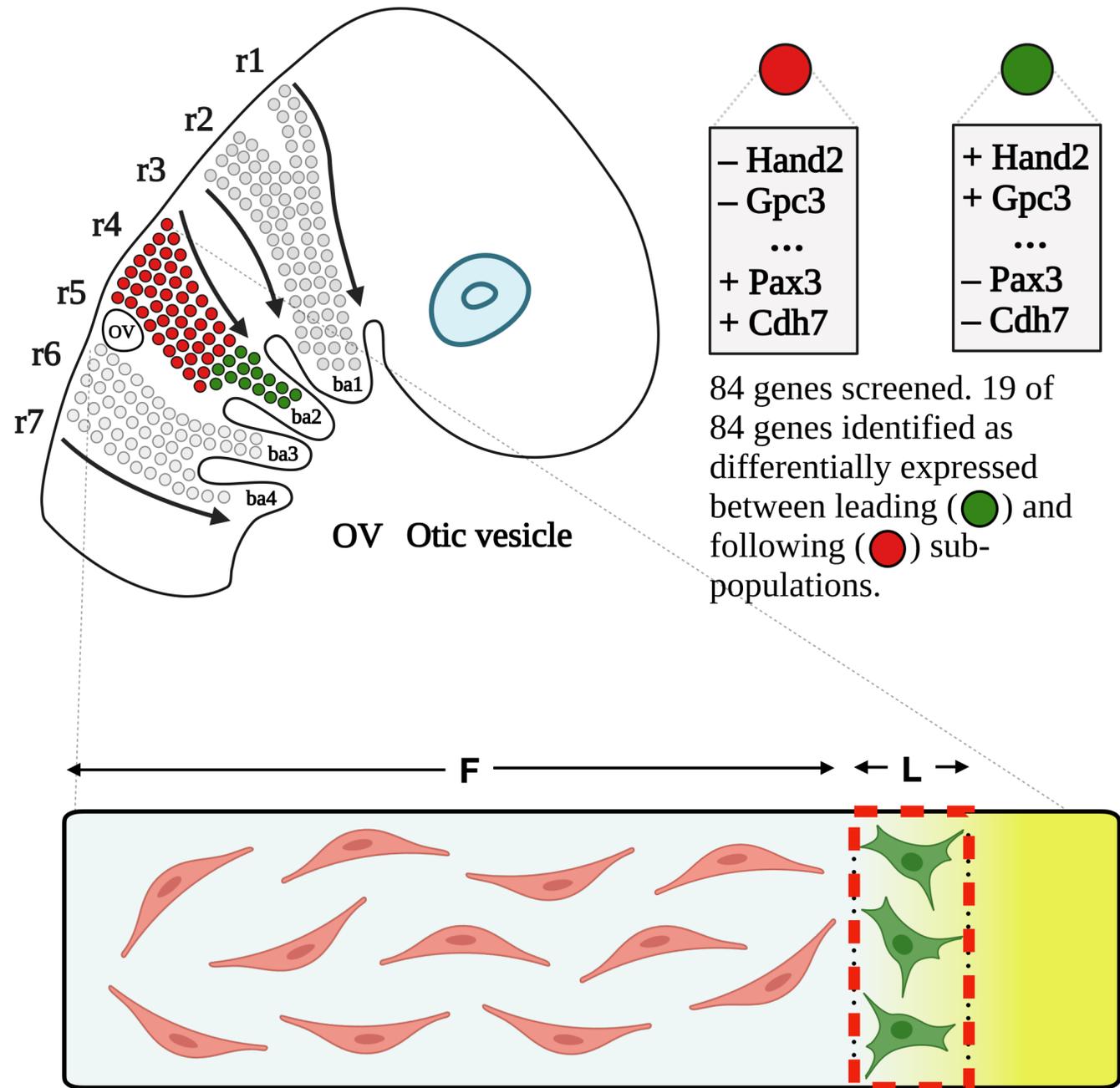
Leader cells -

- **Strong VEGF** gradient exposure.
- **Spiky** shape.

Follower cells -

- **Weak VEGF** gradient exposure.
- **Oblong** shape.

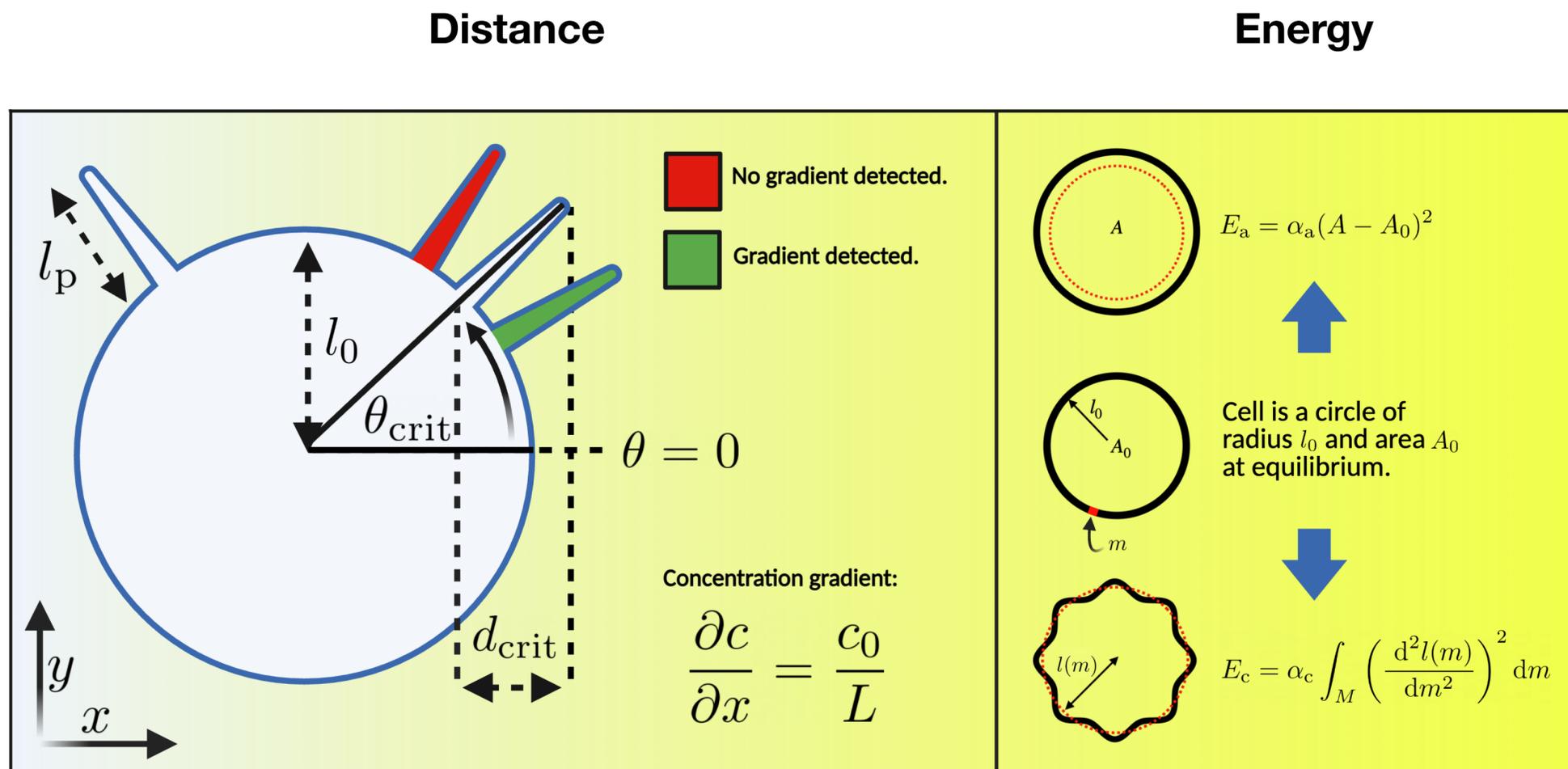
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Cranial neural crest cell morphology

- We developed a mathematical model to understand the **morphological differences** observed across **cranial neural crest streams *in vivo*** [2].
- The model postulates that cranial neural crest cells **adopt morphologies that allow them to find VEGF gradients in the most energetically efficient manner.**



DTER Model

$$\max_{n_p, l_p} \frac{\mathbb{E}(d_x)}{E}(n_p, l_p | \{\gamma_i\})$$

$\{\gamma_i\}$ Environmental parameters

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DTER Model

$$\frac{\mathbb{E}(d_x)}{E} = \frac{g(\beta, \phi) dn_p \int_0^{\theta^{\text{crit}}} f(\theta'') \cos(\theta'') \left(1 - \int_0^{\theta''} f(\theta') d\theta' \right)^{n_p - 1} d\theta''}{n_p l_p^2 + \alpha_a n_p^2 l_p^2 + \alpha_m \beta g(\beta, \phi) d}$$

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- There are **many exciting biological questions** that could be answered with this model.
- We have, thus far, assumed that the number (n_p) and length (l_p) of protrusions extended are **fixed for cells in the model** - there could be many interesting optimal sampling strategies involving a **mixture of protrusion lengths**.
- It is known that **protrusions locally degrade the tissue through which cells migrate**. It would be interesting to explore the existence of **feedback loops between protrusion number (n_p) and tissue stiffness (β)**.
- Cells migrate through **noisy environments** - it would be interesting to study the effect of **noise in VEGF profiles** by embedding this model in an agent-based model of leader-follower neural crest migration.

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Thanks for listening. All questions welcome!



Johnson, S.W., Parsons, M., Baker, R.E. and Maini, P.K., 2025. An energy-based mathematical model of actin-driven protrusions in eukaryotic chemotaxis. *arXiv preprint arXiv:2509.20303*.

References

[1] - Kulesa, P.M., Bailey, C.M., Kasemeier-Kulesa, J.C. and McLennan, R., 2010. Cranial neural crest migration: new rules for an old road. *Developmental Biology*, 344(2), pp.543-554.

[2] - Teddy, J.M. and Kulesa, P.M.. In vivo evidence for short-and long-range cell communication in cranial neural crest cells. *Development*, (2004): 6141-6151.



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