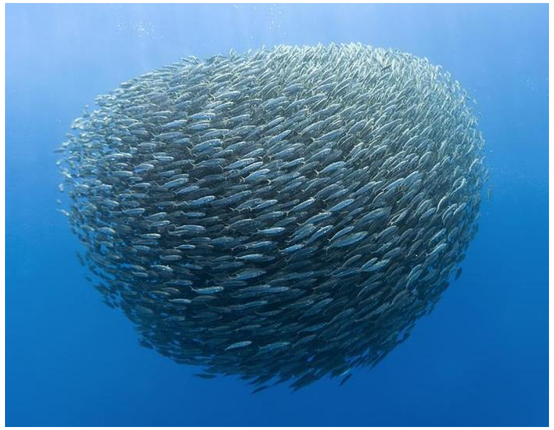
Modelling the dynamics of flapping flyers



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MMSC case study project (HT 2026)

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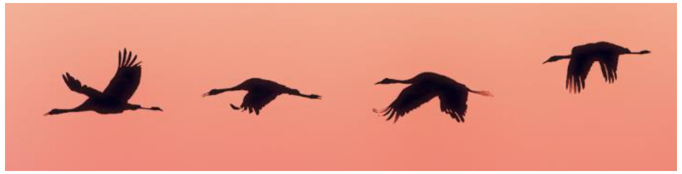
Motivation: birds flying in linear formations

cranes

sandpipers

geese



flamingos

cranes

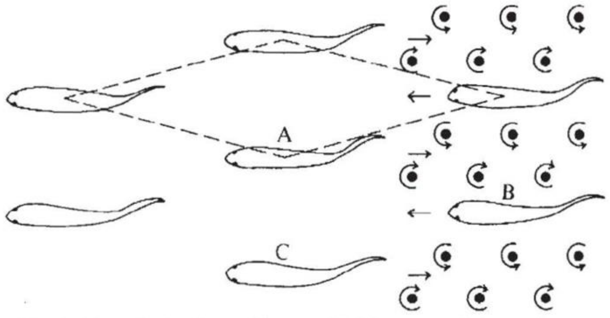
pelicans

Role of flow interactions?

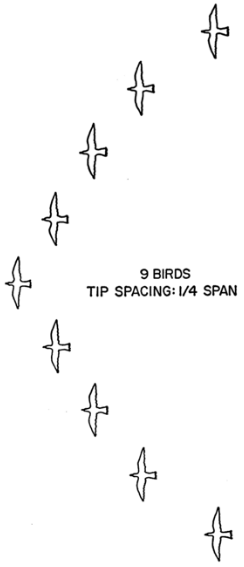
Internal structure and dynamics?

Flock = bird crystal?

Vicsek *et al.*, *Physical Review Letters* (1995)



Weihs, *Nature* (1973)

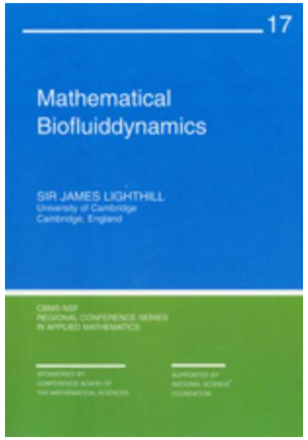


Lissaman & Shollenberger, *Science* (1970)

Sir James Lighthill

We bridge standard models of flocks and schools

"The question arises whether *passive* forces bring the pattern into play or whether very elaborate control mechanisms act to maintain the lattice pattern…"

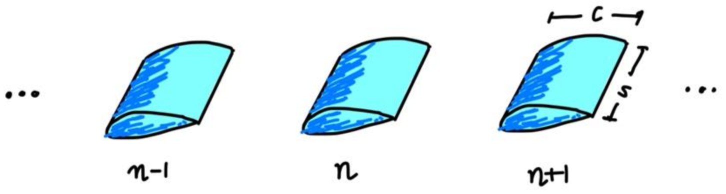


Model ingredients

Dimensions and physical quantities

Flyer parameters

1. Chord (c)
2. Span (s)
3. Mass (M)



Fluid parameters

4. Fluid density (ρ)
5. Viscosity (μ)
6. Wake dissipation rate (τ)

Prescribed kinematic quantities

1. Peak-to-peak flapping amplitude (A)
2. Flapping frequency (f)

One example could be…

Flapping trajectory:

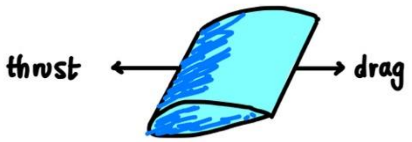
$$\frac{A_n}{2}\sin(2\pi f_n t)$$

Flapping velocity:

$$V_n(t) = \pi f_n A_n \cos(2\pi f_n t)$$

Forces

1. Thrust (T_n)
2. Drag (D_n)



$$\dot{X}_n = U_n$$
$$M\dot{U}_n = T_n - D_n$$

Newton's second law of motion

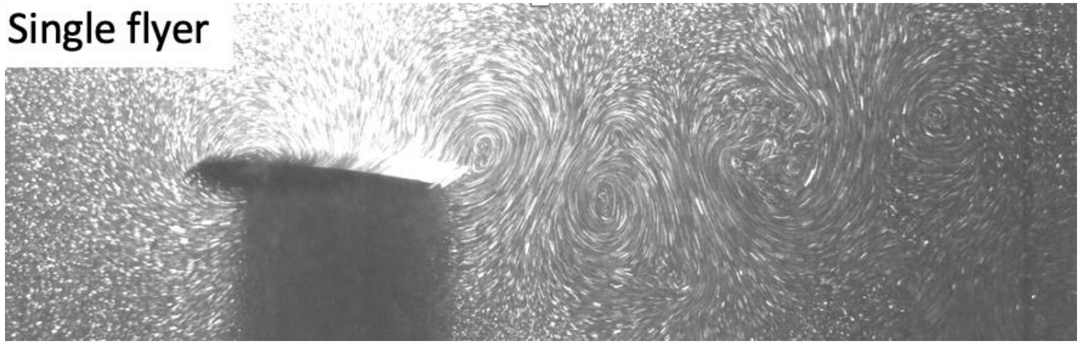
$$T_n = \frac{\rho C_T c s}{2}(\Delta V_n)^2$$

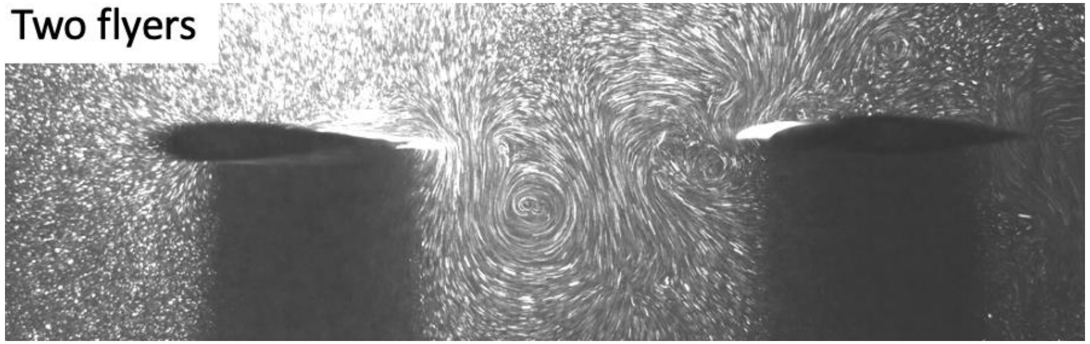
ΔV_n is the relative vertical velocity

The difference between the prescribed flapping velocity and the fluid velocity

Model derivation: wake decay

Flow visualization: vortex interception

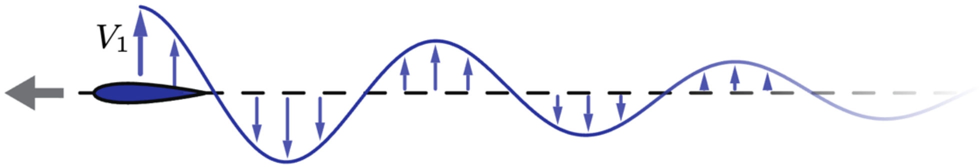




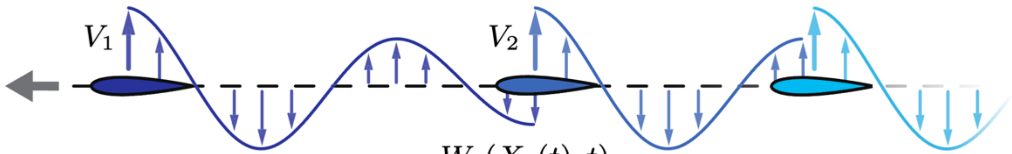
Ramananarivo, Fang, Oza, Zhang & Ristroph, *Phys. Rev. Fluids* (2016)

Delay between the moment when signal is created and when it is experienced by the next flyer
During this time lag, the wake strength is decaying exponentially with a dissipation rate τ

Flyer-wake model



Interaction model

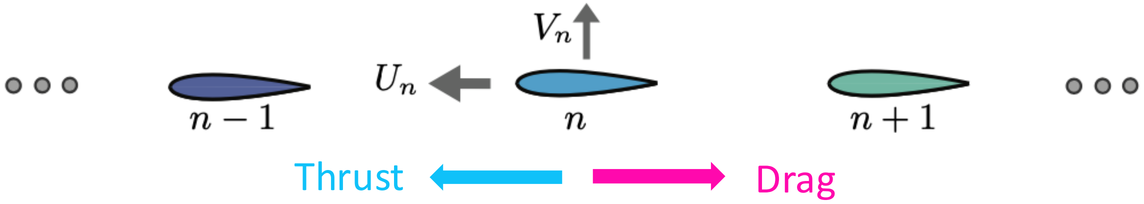


Memory effect: At high Re, flows are long-lived, and information can be stored in the fluid by one flyer and recalled later by another

Model derivation: time delay

How is the effect of **memory** or **delay** included in the model?

$t_n(t)$ is the earlier time when the upstream flyer $(n-1)$ was at the current position of flyer n: $X_{n-1}(t_n(t)) = X_n(t)$



$$\dot{X}_n(t) = U_n(t), \quad n = 1, 2, \ldots, N$$

Newton's 2nd law of motion

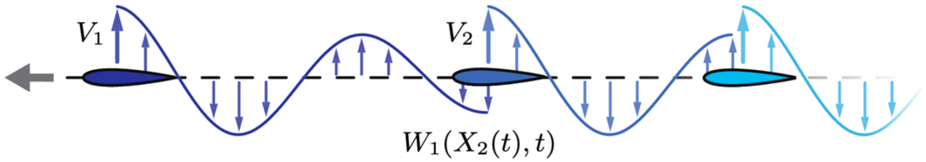
$$\dot{U}_n(t) = \frac{\rho C_T c s}{2M}\left[V_n(t) - V_{n-1}(t_n(t))e^{-(t-t_n(t))/\tau}\right]^2 - \frac{C_D s\sqrt{\rho\mu c}}{2M}U_n^{3/2}(t), \qquad n = 2, \ldots, N$$

Thrust

Skin friction drag

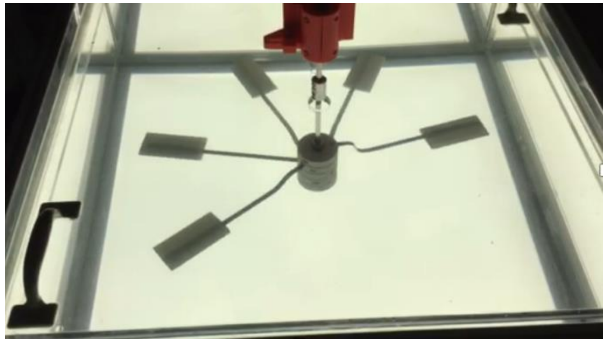
Chain rule on $X_{n-1}(t_n(t)) = X_n(t)$ gives:

$$\dot{t}_n(t) = \frac{U_n(t)}{U_{n-1}(t_n(t))}, \quad n = 1, 2, \ldots, N$$

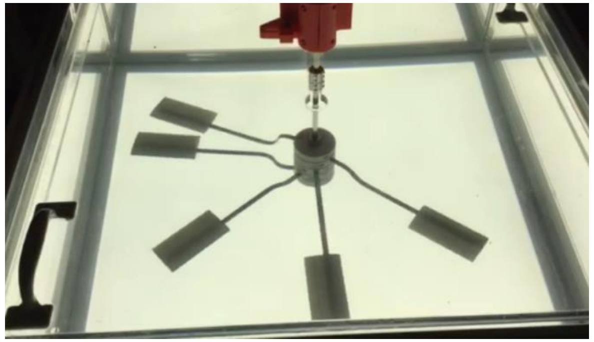


Traveling waves in large groups & *collisions*!

At the beginning…

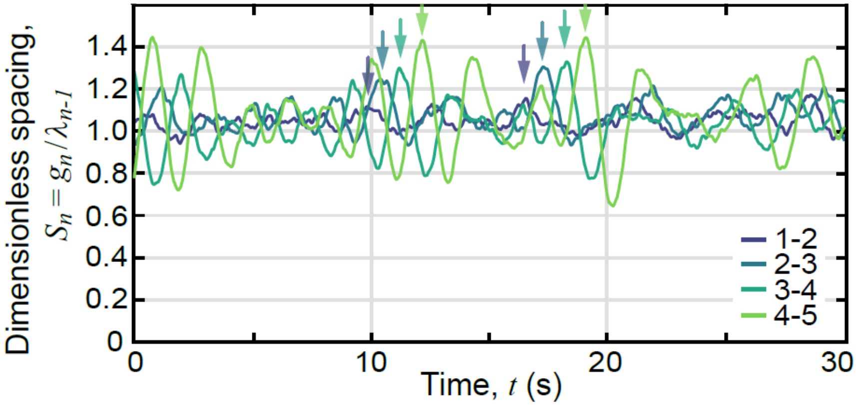


Some time later…

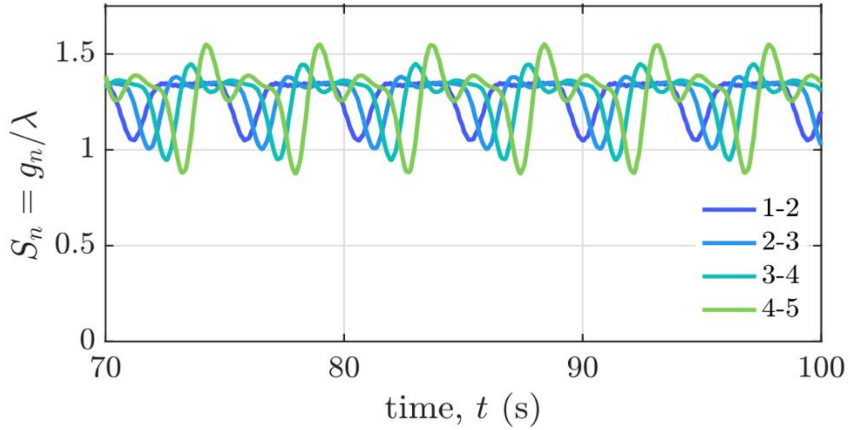


Flonons: Flow-induced fluctuations in flocks of flapping flyers

Motion tracking experimental data

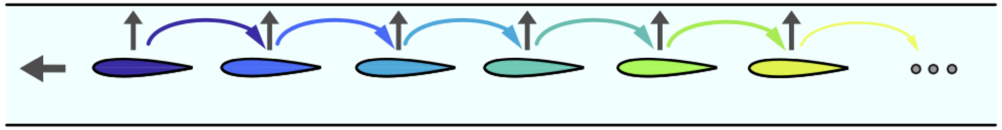


Simulations



Newbolt, Lewis, Bleu, Wu, **Mavroyiakoumou**, Ramananarivo, & Ristroph, *Nat. Commun.* (2024)
Mavroyiakoumou, Wu, & Ristroph, *arXiv:2506.14025* (2025)

Some project goals



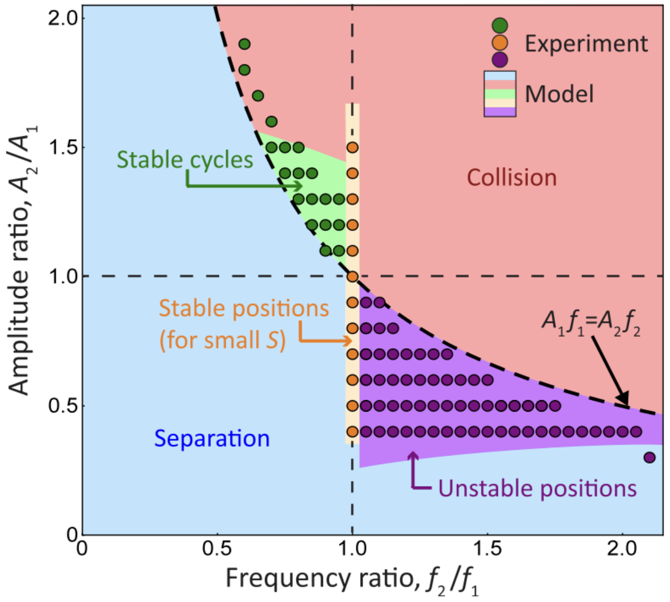
1. Start from the two-flyer system and modify the model such that the flapping kinematics of each flyer are different.

2. Test different active control mechanisms for individual flyers who sense the flow disturbances created by their upstream neighbor.

3. Energetic benefits of different control mechanisms.

4. How quickly is cohesion or stabilization achieved in each case?

5. Increase the number of flyers in the group and repeat the steps above.



Newbolt, Zhang, & Ristroph, *PNAS* (2019)