Introduction to schemes:
Reading topics by week

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Note: The topics listed here for week n will be the subject of a student presentation in the course meeting of week $n+1$. This necessitates a meeting in week 9, which we can hold in-person or remotely, depending on the circumstances.

Week 1 The Spec of a ring, Zariski topology, comparison with classical algebraic geometry.

Week 2 Pre-sheaves and stalks, sheaves, sheafification. The abelian category of sheaves of abelian groups on a topological space. Direct and inverse images of sheaves. Sheaves defined on a topological basis.

Ringed spaces and morphisms of ringed spaces. Affine schemes, construction of the structure sheaf, the equivalence of categories defined by Spec.

Week 3 Schemes, closed subschemes. Global sections. The functor of points.

Properties of schemes: (locally) Noetherian, reduced, irreducible, and integral schemes. Properties of morphisms of schemes: finite type, open/closed immersions, flatness including simple examples of flat families of schemes arising from deformations.

Week 4 Gluing sheaves. Gluing schemes. Affine and projective n-space viewed as schemes.

Products, coproducts and fiber products in category theory. Existence of products of schemes. Fibers and preimages of morphisms of schemes. Base change.

Week 5 Further properties of morphisms of schemes: separated, universally closed, and proper morphisms. Projective n-space and projective morphisms. Abstract varieties. Complete varieties. Scheme structure on a closed subset of a scheme.

Week 6 Sheaves of modules. Vector bundles and coherent sheaves. The abelian category of sheaves of modules over a scheme. Pull-backs.

Quasi-coherent sheaves. Gluing sheaves of modules. Classification of (quasi-)coherent sheaves on Spec of a ring.

Week 7 Čech cohomology. Vanishing of higher cohomology groups of quasi-coherent sheaves on affine schemes. Independence of Čech cohomology on the choice of open cover. Line bundles, examples on projective n-space.

Week 8 Sheaf cohomology. Acyclic resolutions. Comparison of sheaf cohomology and Čech cohomology.

Quasi-coherent sheaves on projective n-space, graded modules, and Proj of a graded ring.