$$p(A_n, t+dt) - p(A_n,t) =$$

$$\frac{P_m}{2} dt \left[p(A_{n-1}, O_n, t) - p(O_{n-1}, A_n, t) \right]$$

$$+ \frac{P_m}{2} dt \left[p(O_n, A_{n+1}, t) - p(A_n, O_{n+1}, t) \right]$$

$$+ \frac{P_m}{2} \rho dt \left[p(A_{n-1}, O_n, t) + p(O_{n-1}, A_n, t) \right]$$

$$- \frac{P_m}{2} \rho dt \left[p(O_n, A_{n+1}, t) + p(A_n, O_{n+1}, t) \right]$$

$$\therefore \frac{p(A_n, t+dt) - p(A_n,t)}{dt} = \frac{P_m}{2} \Big[p(A_{n-1},t) - p(A_{n-1}, A_n, t) - \{ p(A_n,t) - p(A_{n-1}, A_n, t) \} + p(A_{n+1},t) - p(A_n, A_{n+1}, t) - \{ p(A_n,t) - p(A_n, A_{n+1}, t) \} \Big]$$

$$+ \rho \{ \qquad \}$$

$$\frac{P_m}{2} \left[p(A_{n-1},t) - 2p(A_n,t) + p(A_{n+1},t) \right]$$

We cannot do the same for the $\rho \{ \qquad \}$ term because we would obtain

$$\rho \frac{P_m}{2} \Big[p(A_{n-1},t) - p(A_{n-1}, A_n, t) + p(A_n,t) - p(A_{n-1}, A_n, t) - \{ p(A_{n+1},t) - p(A_n, A_{n+1}, t) + p(A_n,t) - p(A_n, A_{n+1}, t) \} \Big]$$

$$= \rho \frac{P_m}{2} \left[p(A_{n-1},t) - p(A_{n+1},t) - 2p(A_{n-1}, A_n, t) + 2p(A_n, A_{n+1}, t) \right]$$

Now we see that we have terms $p(A_{n-1}, A_n, t)$, $p(A_n, A_{n+1}, t)$ so we need to use the ~~also~~ independent assumption

$$p(A_n, A_{n+1}, t) = p(A_n, t)\,p(A_{n+1}, t)$$

etc.

Hence, we obtain

$$\frac{P_m}{2}\rho\Big[\,p(A_{n-1},t) - p(A_{n+1},t) - 2\,p(A_{n-1},t)\,p(A_n,t) + 2\,p(A_n,t)\,p(A_{n+1},t)\Big]$$

$[\quad]$ we can write as

$$\big[p(A_{n-1},t) - p(A_{n+1},t)\big]\big[1 - p(A_n,t)\big] + p(A_n,t)\big[p(A_{n+1},t) - p(A_{n-1},t)\big]$$

Now write $p(A_n, t+dt) = p(\underbrace{n\,dx}_{x}, t+dt)$ etc.

$\therefore$ the ρ term becomes

$$\underbrace{\big[p(x-dx,t) - p(x+dx,t)\big]}\big[1 - p(x,t)\big] + p(x,t)\underbrace{\big[p(x+dx,t) - p(x-dx,t)\big]}$$

Note: we can write this as $[\quad][1-2p(x,t)]$

ie $$\left(\cancel{p(x)} - \frac{\partial p}{\partial x} dx + \cancel{\frac{\partial^2 p}{\partial x^2} \frac{dx^2}{2}} + \cdots \\ \cancel{- p(x)} - \frac{\partial p}{\partial x} dx - \cancel{\frac{\partial^2 p}{\partial x^2} \frac{dx^2}{2}} + \cdots \right) \left(1 - 2p(x,t)\right)$$

ie $$\cancel{+ p(x,t)} \quad \frac{\partial p}{\partial x} \left(1 - 2p(x,t)\right) dx$$

which is the same as

$$\frac{\partial}{\partial x}\left(p(1-p)\right) dx$$

$$\left[\frac{\partial}{\partial x}\left(p(1-p)\right) = \frac{\partial}{\partial x}\left(p - p^2\right) = (1-2p)\frac{\partial p}{\partial x} \right]$$