BO1.1. History of Mathematics
Lecture II
Dissemination and development
(600 BC – AD 1600)

MT26 Week 1

Summary

- Influence of the ancient world
- The European Renaissance (15th and 16th centuries)
- Rediscovery and transmission of ancient texts
- The 16th century
- A case study: Napier's invention of logarithms 1614

Ancient influences on early modern European mathematics

(Early modern = roughly 1400–1800)

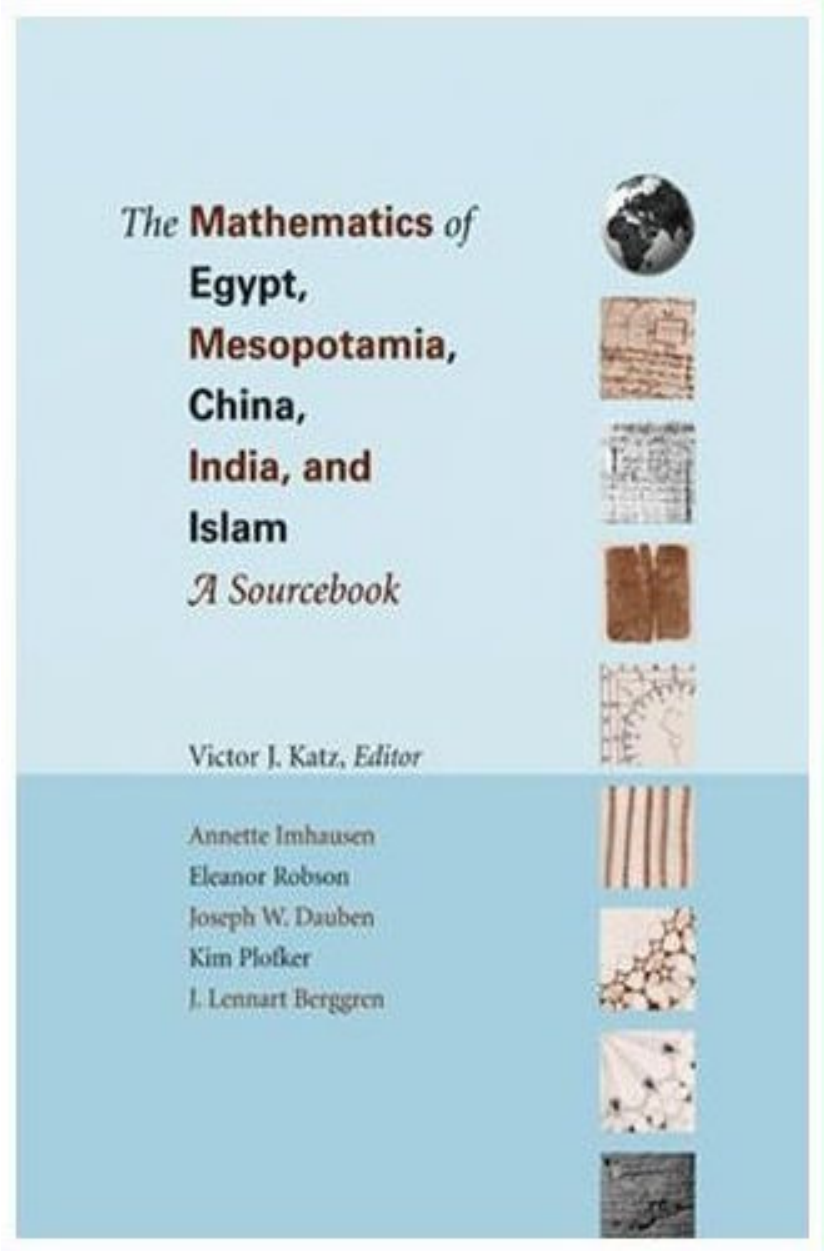
The prior mathematical accomplishments of

- China
- India
- Mesopotamia
- Egypt
- and most other places

were largely unknown in Europe until the 19th century

The single greatest influence on early modern European mathematics was the mathematics of ancient Greece, with that of mediaeval Islam (a little later) coming a close second

A sourcebook on the ancient origins of mathematics



Ancient Greek mathematics

Earliest origins of Greek mathematics in 6th century BC

But what do we mean by 'Greek'?

500 BC – 300 BC	Collection of city-states in Greece
300 BC – AD 500	Greek-speaking peoples around the Mediterranean, especially in Alexandria

Some major figures of 'Greek' mathematics

Pythagoras	Samos (Greece)?	c. 600 BC
Euclid	Alexandria (Egypt)?	c. 300 (or 250?) BC
Archimedes	Syracuse (Sicily)	c. 250 BC
Apollonius	Perga (Turkey)	c. 180 BC
Diophantus	Alexandria (Egypt)	c. AD 200

Euclid's *Elements*

The 'elements of geometry' in 13 books, compiled around 300 (250?) BC from existing geometrical knowledge

Books I–VI	plane geometry	points, lines, angles, circles, ...
Books VII–X	properties of numbers	odd, even, square, triangular, prime, perfect, ...
Books XI–XIII	solid geometry	cube, tetrahedron, icosahedron, ...

David Joyce's online version of Euclid's *Elements*

Oliver Byrne's colour version of the first six books

Euclid's *Elements*, Book I

23 definitions: point, line, surface, angle, circle, ...

5 postulates: what one can do

e.g. a straight line may be drawn between any two points;
a circle may be drawn with given centre and radius

5 'common notions': how one may reason

e.g. if equals are added to equals, then the wholes are equal

48 propositions: each built only on what has gone before

The influence of Euclid's *Elements*



HUGE influence on Western mathematics:

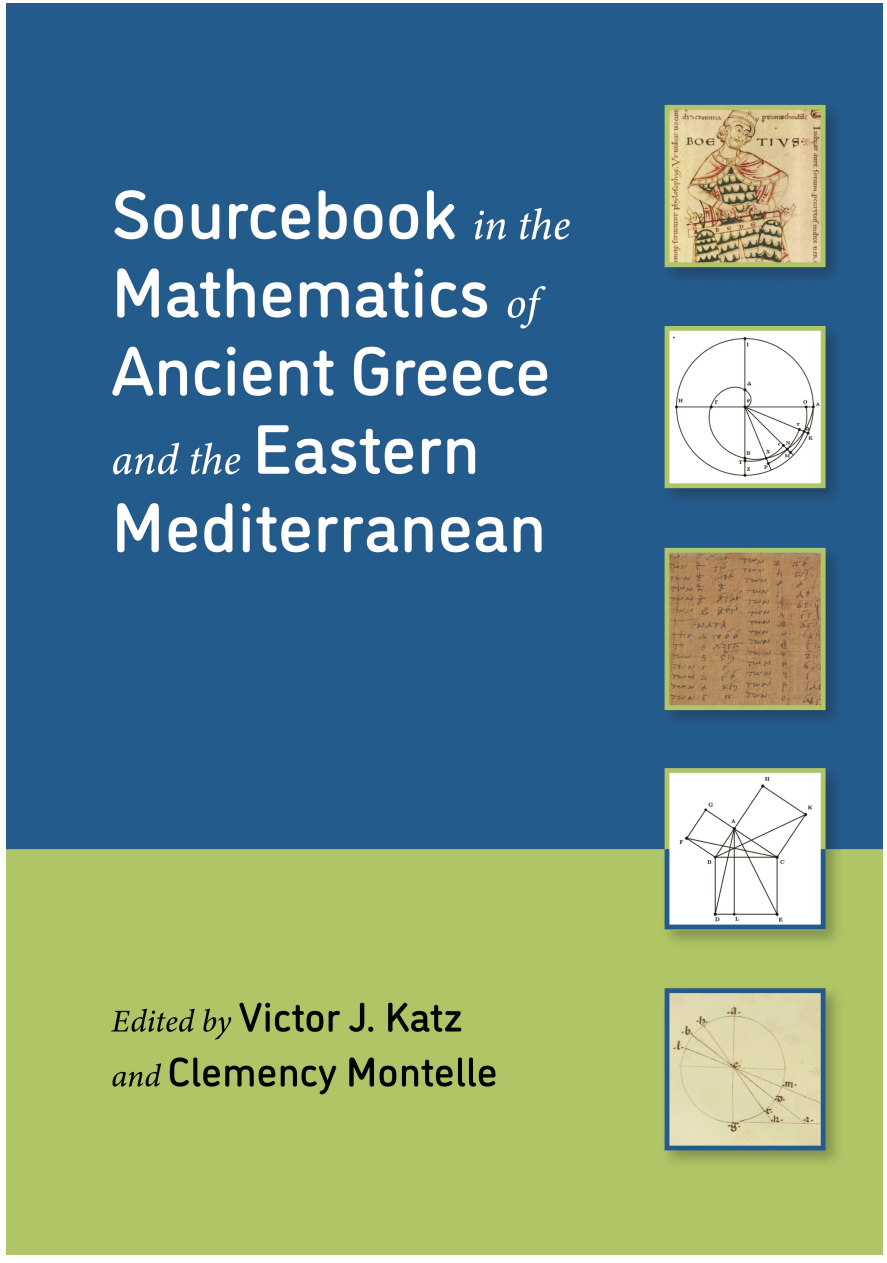
- hundreds of editions and translations from renaissance onwards
- basis of mathematics teaching in schools until c. 1960
- style: definitions—axioms—theorems—proofs
- status of 'Parallel Postulate' led to much investigation and, ultimately, non-Euclidean geometries
- problems of 'ruler and compass' construction inspired much investigation and many new discoveries
- wider cultural importance: https://www.history.ox.ac.uk/reading-euclid

Other Greek authors

Archimedes	d. 212 BC:	method of exhaustion and much else
Apollonius	c. 180 BC:	conic sections
Diophantus	c. AD 250:	*Arithmetica* in 13 books (number problems)

Also had HUGE influence on Western mathematics

A sourcebook on ancient Greek mathematics



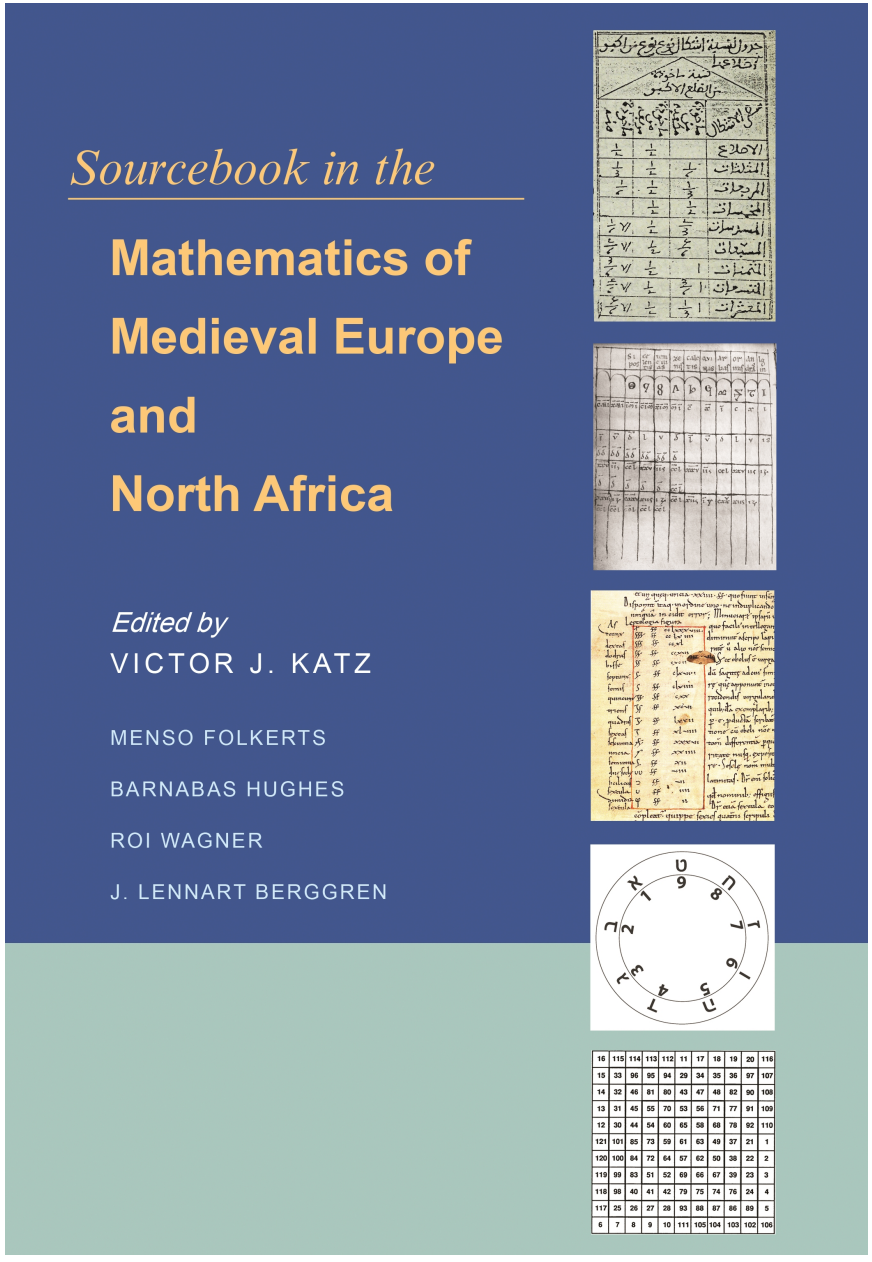
Remnants of the collapse of the ancient world

in Greek:	manuscripts preserved at Constantinople and in libraries or collections around the Mediterranean
in Latin:	writings by Boethius (c. 480–524) on philosophy, arithmetic, geometry, music

The spread of Islam and Islamic learning

632–732:	Islam spreads throughout Middle East, north Africa, and into Spain and Portugal
c. 820:	*Bayt al-Ḥikma*, the House of Wisdom, founded in Baghdad under Caliph al-Ma'mūn; it became a centre for translation into Arabic from Greek, Persian, Sanskrit
c. 825:	al-Khwārizmī active in Baghdad
9th century:	texts on arithmetic, algebra, astronomy reach Spain
12th century:	translations from Arabic to Latin

A sourcebook on mediaeval mathematics



The mid-Renaissance (15th and 16th centuries)

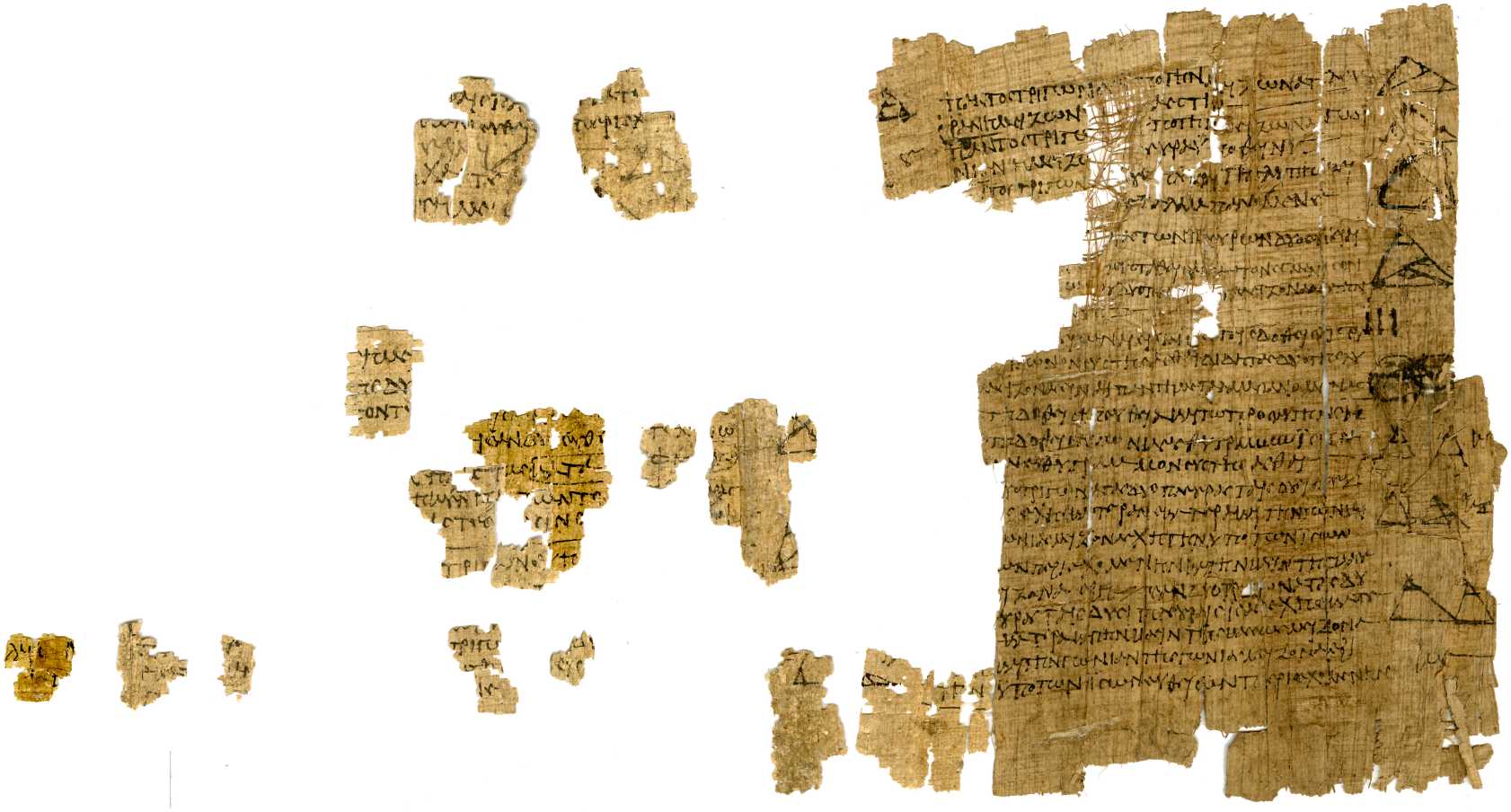
Classical mathematical texts more widely available due to:

- rediscovery of manuscripts
- revival of knowledge of Greek
- transmission of otherwise lost texts via Arabic versions
- (Western) invention of printing (Gutenberg, c. 1436)

Euclid's *Elements*: transmission history

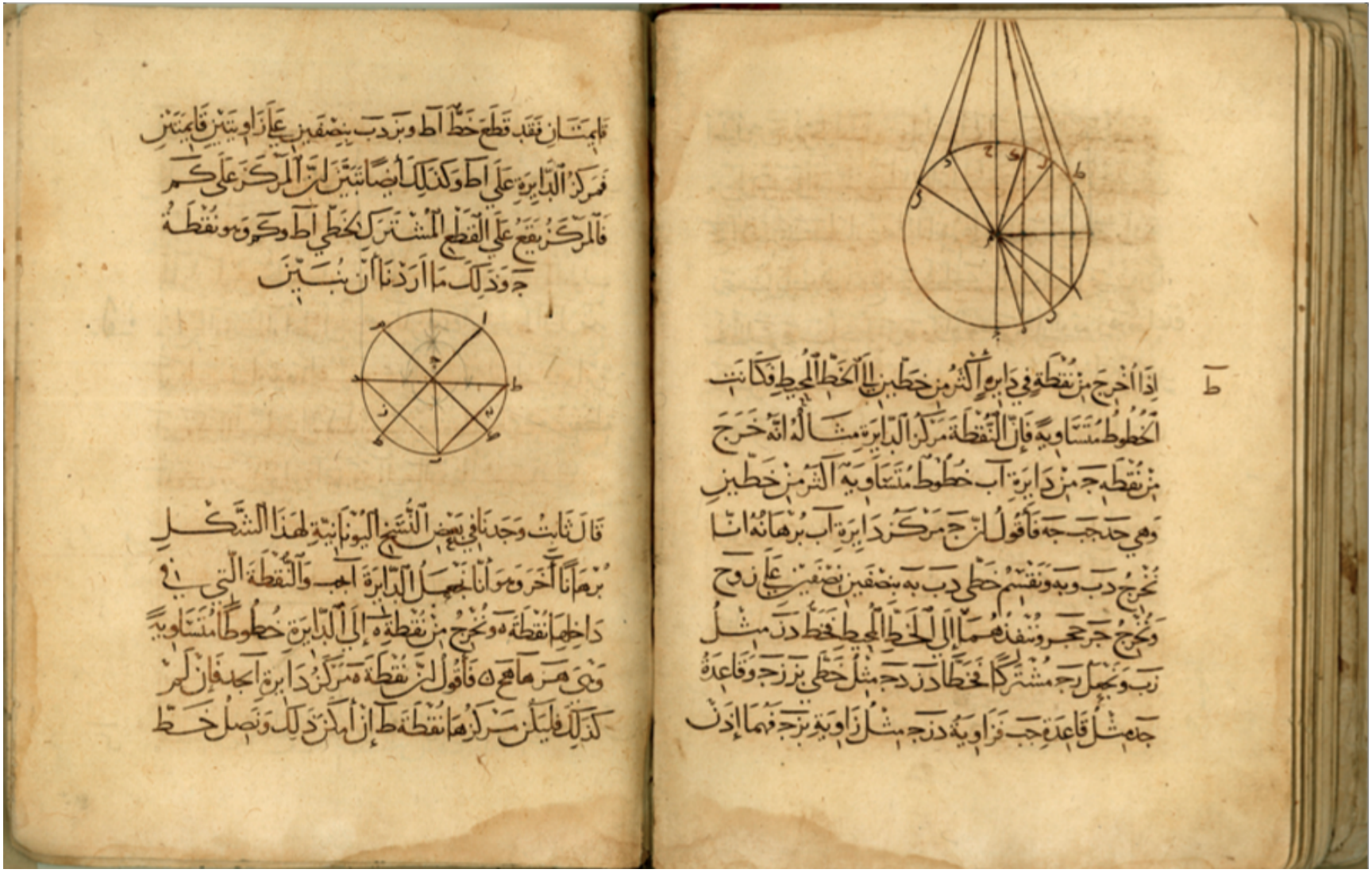
- commentaries written by Pappus (c. AD 320), Theon (c. AD 380), Proclus (c. AD 450)
- a few propositions in Boethius (c. AD 500)
- copies in Greek (earliest from Constantinople, AD 888)
- many translations or commentaries in Arabic (AD 750–1250)
- mediaeval translations from Arabic to Latin: Adelard of Bath (1130), Robert of Chester (1145), Gerard of Cremona (mid-12th century)
- printed editions in Latin or Greek from 1482 onwards

Early fragments of Euclid



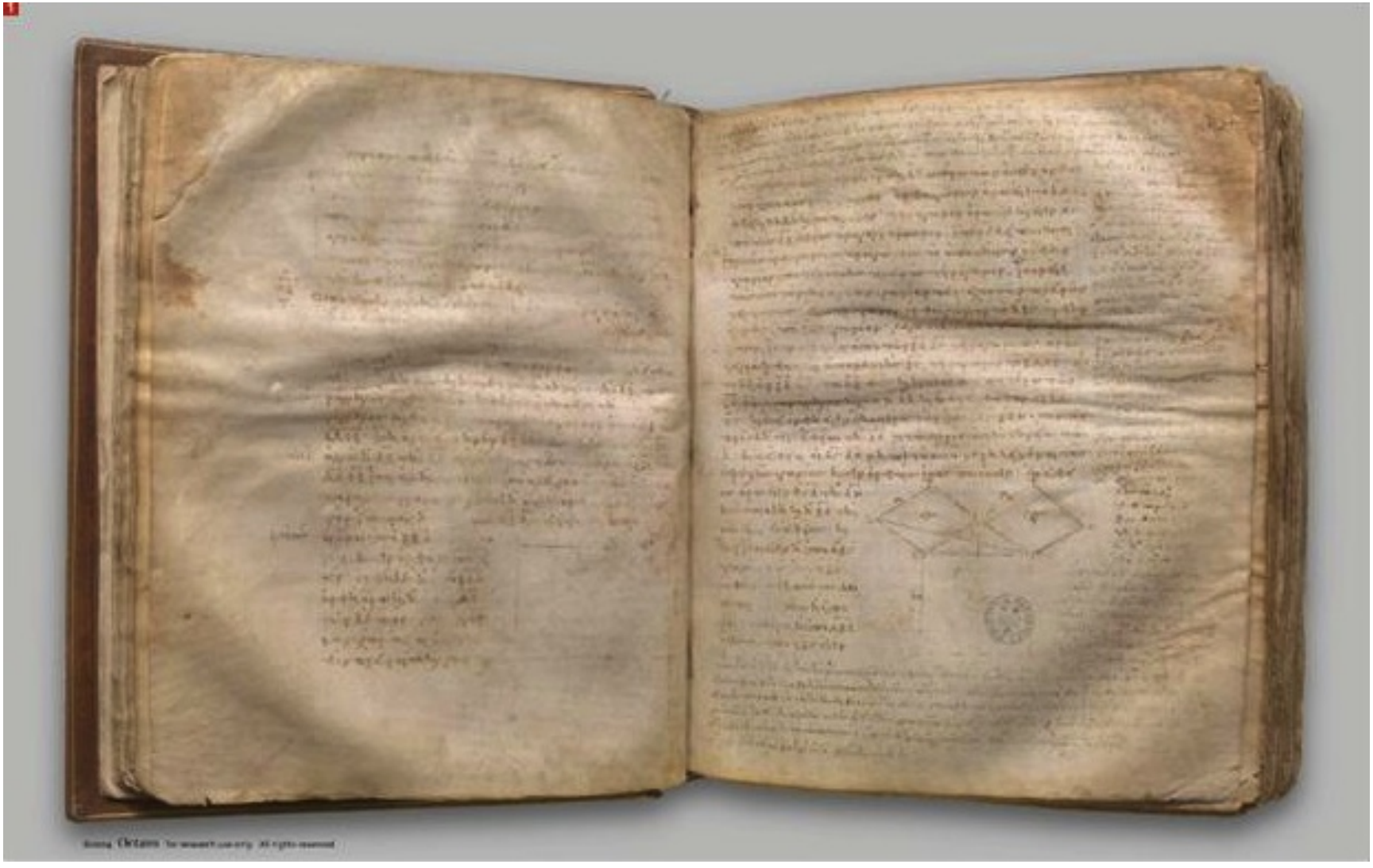
Fragments of Book I among the Oxyrhynchus Papyri (3rd century)

Euclid in Arabic



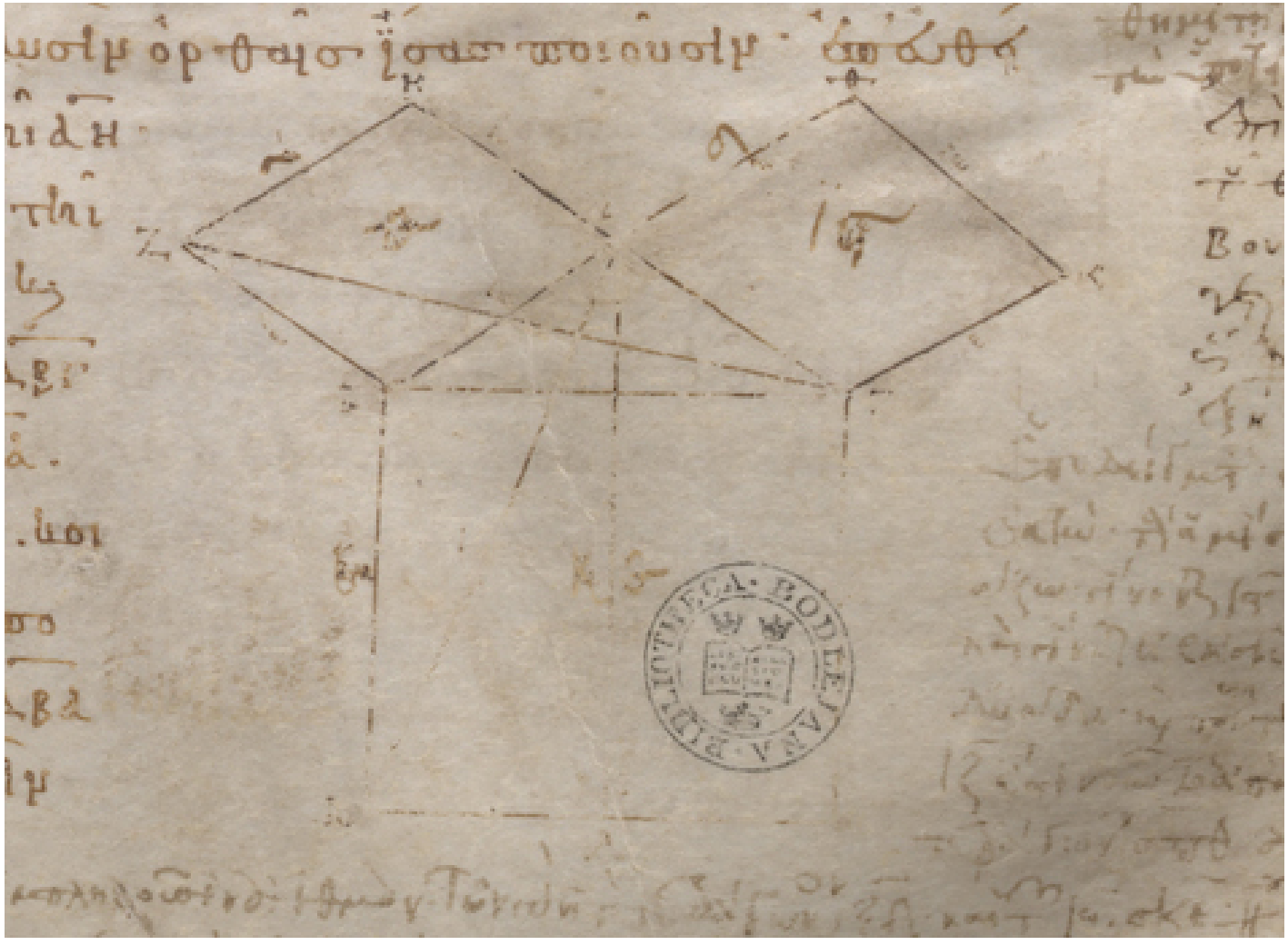
Translated from the Greek by Ishaq ibn Hunayn, AD 1466

Euclid I.47 from Bodleian MS dated 888



Whole manuscript is digitised:
http://www.claymath.org/library/historical/euclid/

Euclid I.47 from Bodleian MS dated 888



http://www.claymath.org/library/historical/euclid/files/elem.1.47.html

Treatises by Archimedes: transmission history

- quoted or explained by Pappus (c. 320 AD), Theon (c. 380 AD), Eutocius (c. 520 AD)
- 6th-century Byzantine 'collected works' (Isidore of Miletus)
- several translations of individual treatises into Arabic
- translations from Arabic into Latin
- a new find in the twentieth century: www.archimedespalimpsest.org

Apollonius' *Conics* (c. 180 BC): transmission history

- Books I–IV survived in Greek
- Books V–VII survived only in Arabic
- Book VIII is lost, known only from commentaries
- early (Latin) printed edition, 1566

(See: *Mathematics emerging*, §1.2.4.)

16th century change

New forces at work in the 16th century:

- global exploration
- growth of international commerce
- new technology (in printing, shipping, military engineering, instrumentation, etc.)

16th century change

New ideas emerging in mathematics:

- Solution of cubic and quartic equations (Tartaglia, Cardano, Ferrari — see Lecture IX)
- New conceptions of number, especially in Simon Stevin's *L'arithmétique* (1585): discrete and continuous quantities united, negative numbers accepted (but not imaginaries — see Lecture XIII)
- Emergence of symbolic notation — see Lecture III

A case study of a text from 1614

Napier's invention of logarithms:

- what did 17th-century mathematics look like?
- how can we begin to read historical texts?

Napier's definition of a logarithm (of a sine)

The **Logarithme** *therefore of any sine is a number very neerely expressing the line, which increased equally in the meane time, whiles the line of the whole sine decreased proportionally into that sine, both motions being equal-timed, and the beginning equally swift.*

?

Context, content, significance

Context:	who? when? where? why?
Content:	what is it about? how is it written?
Significance:	why did/does it matter?
Historical Significance:	what new insight does this text offer us?

Context — who?

John Napier (1550–1617), Merchiston, Scotland

Scottish landowner with interests in:

- mining
- calculating aids
- astrology/astronomy
- The Book of Revelation



See *Oxford Dictionary of National Biography*:
http://www.oxforddnb.com/view/article/19758

Context — why?

From Napier's preface to the English translation of 1616:

> *Seeing there is nothing (right well-beloved Students of the Mathematics) that is so troublesome to mathematical practice, nor that doth more molest and hinder calculators, than the multiplications, divisions, square and cubical extractions of great numbers, which besides the tedious expense of time are for the most part subject to many slippery errors, I began therefore to consider in my mind by what certain and ready art I might remove those hindrances.*

Context — why?

Inspired by the 16th-century technique of prosthaphaeresis:

the use of trigonometric identities such as

$$\cos x \cos y = \frac{1}{2}\left[\cos(x+y) + \cos(x-y)\right]$$

$$\sin x \sin y = \frac{1}{2}\left[\cos(x-y) - \cos(x+y)\right]$$

to convert multiplication into addition.

Context — in what form, and in which language?

Original Latin text of 1614:

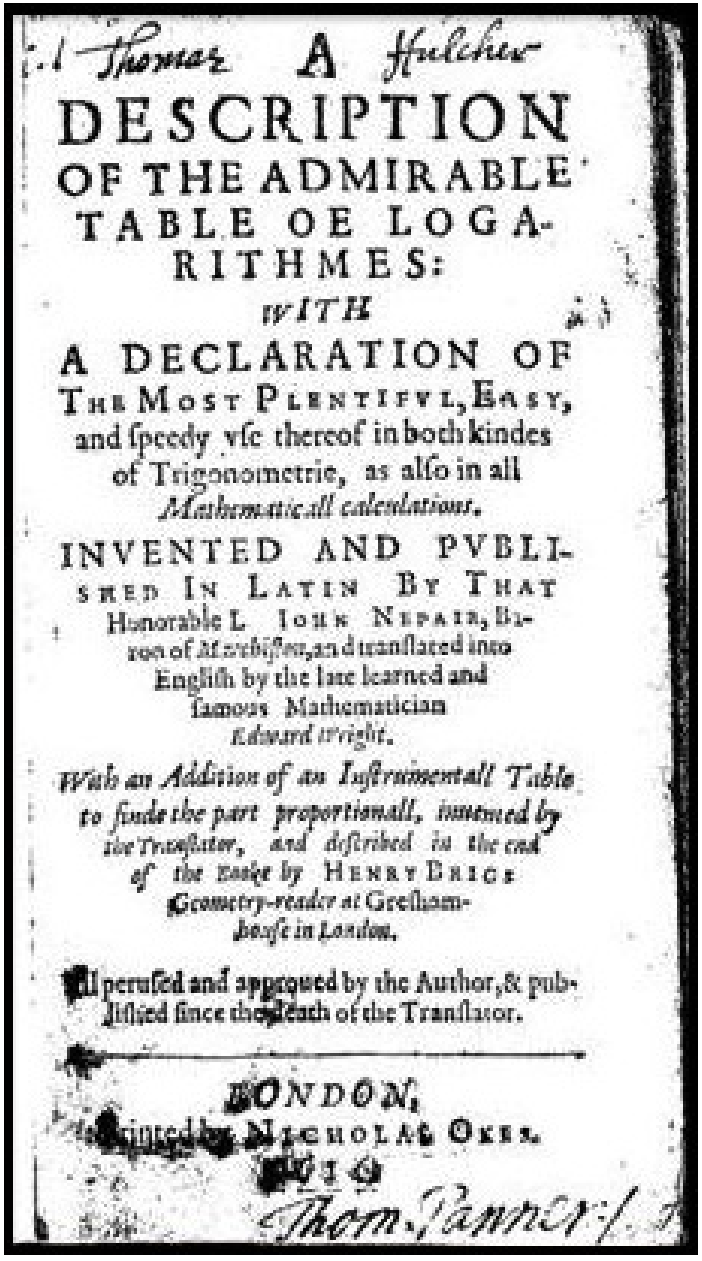
Mirifici logarithmorum canonis descriptio

translated into English by Edward Wright in 1616 as

A description of the admirable table of logarithms

Both the Latin original and English translation are available online

Napier's 1616 title-page decoded



A DESCRIPTION
OF THE ADMIRABLE
TABLE OE LOGA-
RITHMES:
WITH
A DECLARATION OF
THE MOST PLENTIFVL, EASY,
and speedy vse thereof in both kindes
of Trigonometrie, as also in all
Mathematicall calculations.
INVENTED AND PVBLI-
SHED IN LATIN BY THAT
Honorable L IOHN NEPAIR, Ba-
ron of Merchiston, and translated into
English by the late learned and
famous Mathematician
Edward Wright.
With an Addition of an Instrumentall Table
to finde the part proportionall, invented by
the Translator, and described in the end
of the Booke by HENRY BRIGS
Geometry-reader at Gresham-
house in London.
All perused and approued by the Author, & pub-
lished since the death of the Translator.
LONDON,
Printed by NICHOLAS OKES.
1616
Thom. Panner

Inventor:
- John Napier (1550–1617)

Translator:
- Edward Wright (?1558–1615) (interests: navigation, charts and tables)

Additional material:
- Henry Briggs (1561–1630) Gresham Professor of Geometry, later Savilian Professor of Geometry at Oxford (interests: navigation)

Printer:
- Nicholas Okes

Readers:
- Thomas Hulcher,
- Thomas Panner

Napier's logarithms: content

Recall:

> *The* **Logarithme** *therefore of any sine is a number very neerely expressing the line, which increased equally in the meane time, whiles the line of the whole sine decreased proportionally into that sine, both motions being equal-timed, and the beginning equally swift.*

Napier's logarithms

peare by the 19 *Prop.* 5. and 11. *Prop.* 7, *Euclid.*

3 *Def.* *Surd quantities, or vnexplicable by number, are said to be defined, or expressed by numbers very neere, when they are defined or expressed by great numbers which differ not so much as one vnite from the true value of the Surd quantities.*

As for example. Let the semidiameter, or whole sine be the rational number 10000000 the sine of 45 degrees shall be the square root of 50,000,000,000,000, which is surd, or irrationall and inexplicable by any number, & is included between the limits of 7071067 the lesse, and 7071068 the greater: therfore, it differeth not an vnite from either of these. Therefore that surd sine of 45 degrees, is said to be defined and expressed very neere, when it is expressed by the whole numbers, 7071067, or 7071068, not regarding the fractions. For in great numbers there ariseth no sensible error, by neglecting the fragments, or parts of an vnite.

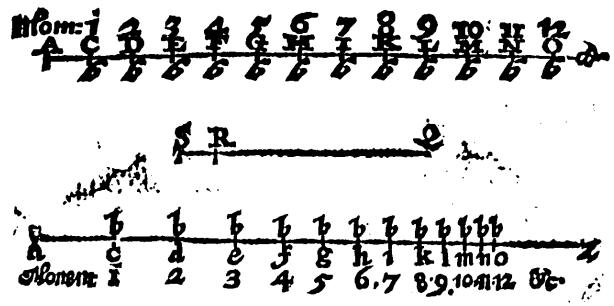
4 *Def.* *Equall-timed motions are those which are made together, and in the same time.*

As in the figures following, admit that B be moued from A to C, in the same time, wherin *b* is moued from *a* to *c* the right lines A C & *a c*, shall be sayd to be described with an equall-timed motion.

5 *Def.* *Seeing that there may bee a slower and a swifter motion giuen then any motion, it shall necessarily follow, that there may be a motion giuen of equall swiftnesse to any motion (which wee define to be neither swifter nor slower.)*

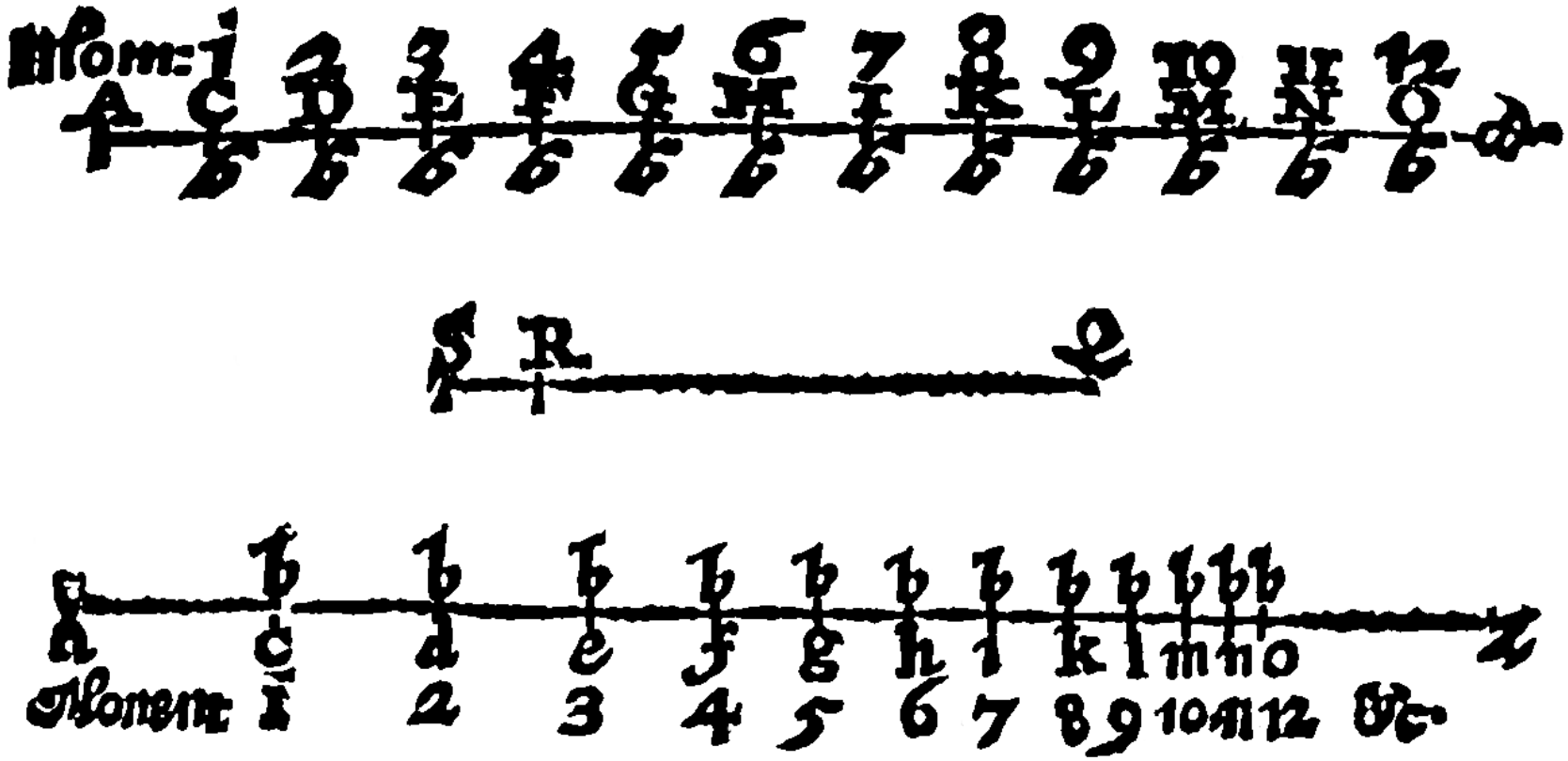
6 *Def.* The Logarithme *therfore of any sine is a number very neerely expressing the line, which increased*

sed equally in the meane time, whiles the line of the whole sine decreased proportionally into that sine, both motions being equal-timed, and the beginning equally swift.

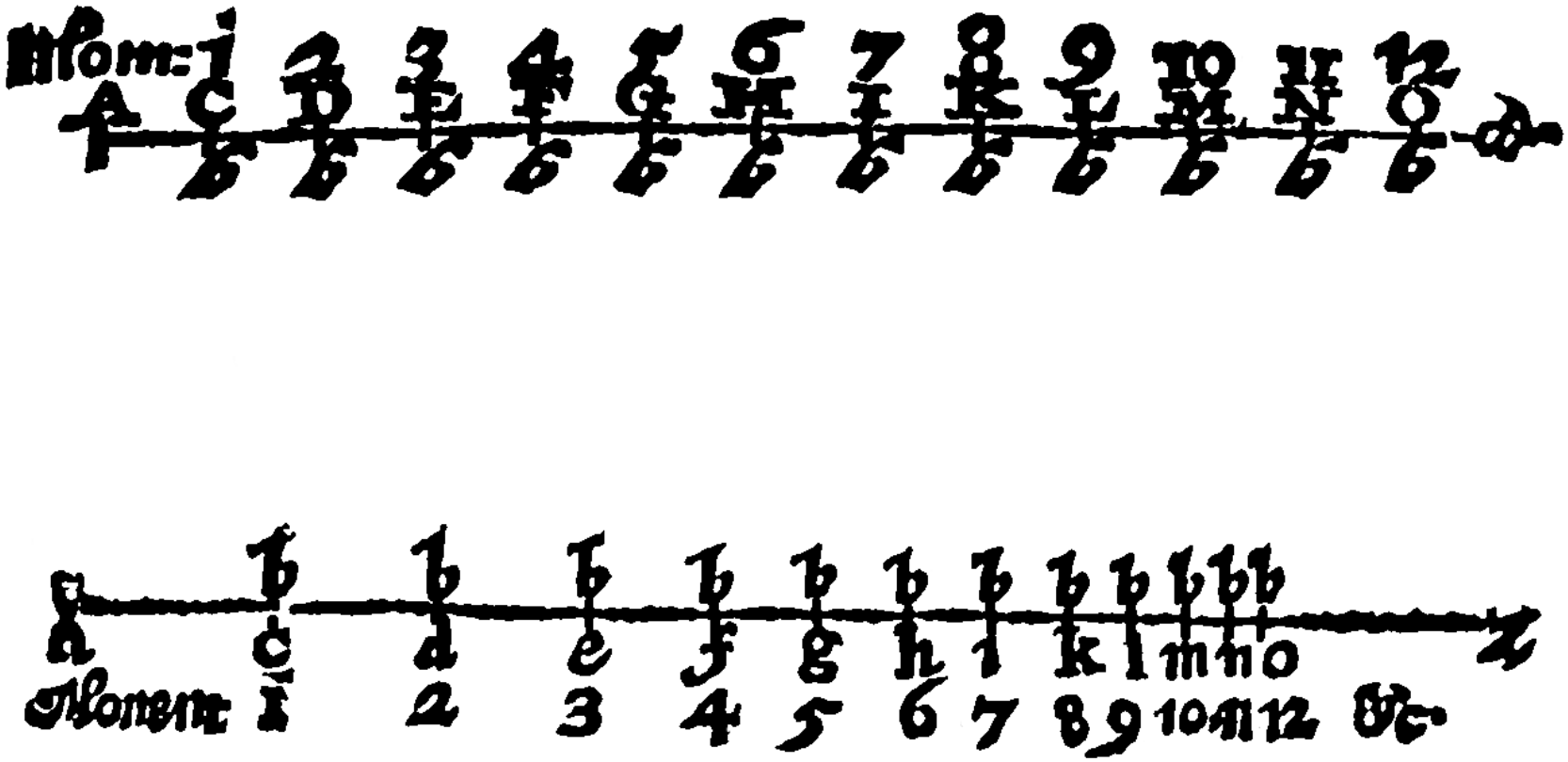


As for example. Let the 2 figures going afore bee here repeated, and let B bee moued alwayes, and euery where with equall, or the same swiftnesse wherewith *b* beganne to bee moued in the beginning, when it was in *a*. Then in the first moment let B proceed from A to C, and in the same time let *b* moue proportionally from *a* to *c*, the number defining or expressing A C shal be the *Logarithme* of the line, or sine *c* Z. Then in the second moment let B bee moued forward from C to D. And in the same moment or time let *b* be moued proportionally from *c* to *d*, the number defininig A D, shall bee the *Logarithme* of the sine *d* Z. So in the third moment let B go forward equally from D to E, and in the same moment let *b* be moued forward proportionally from *d* to *e*, the number expressing A E the *Logarithme* of the sine *e* Z. Also in the fourth moment, let B proceed

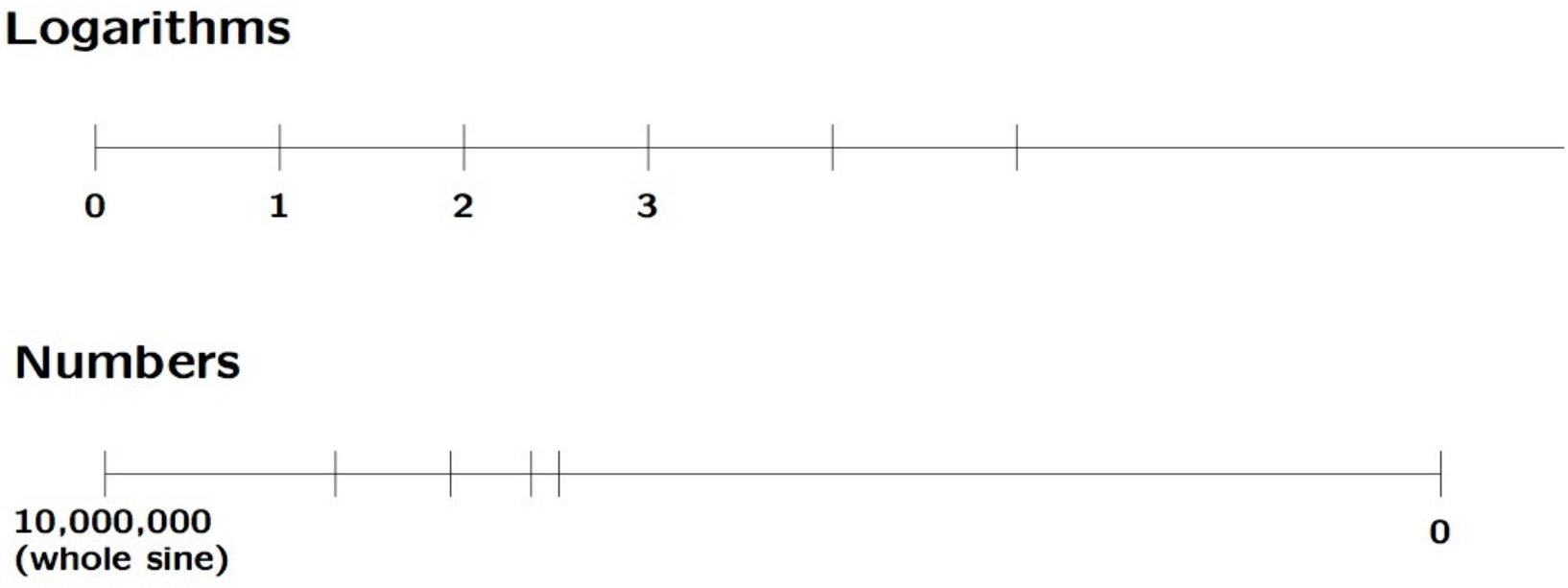
Napier's logarithms



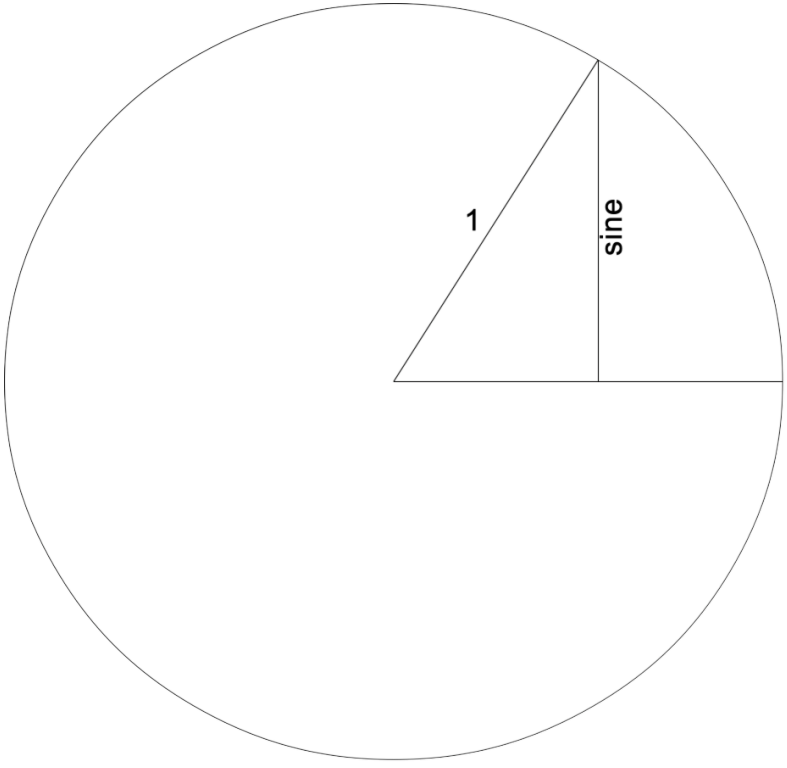
Napier's logarithms



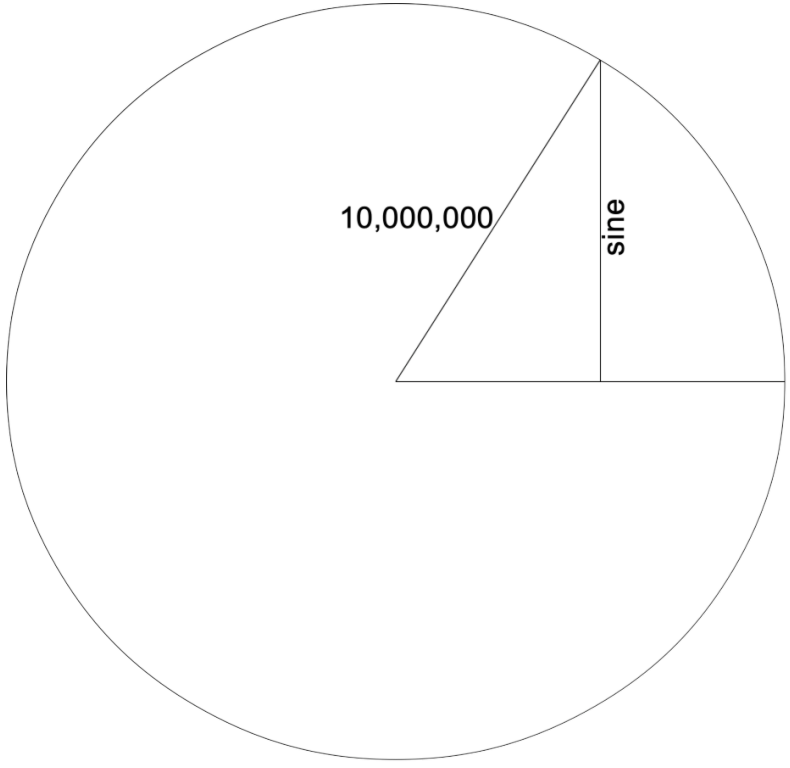
Napier's logarithms



Napier's logarithms

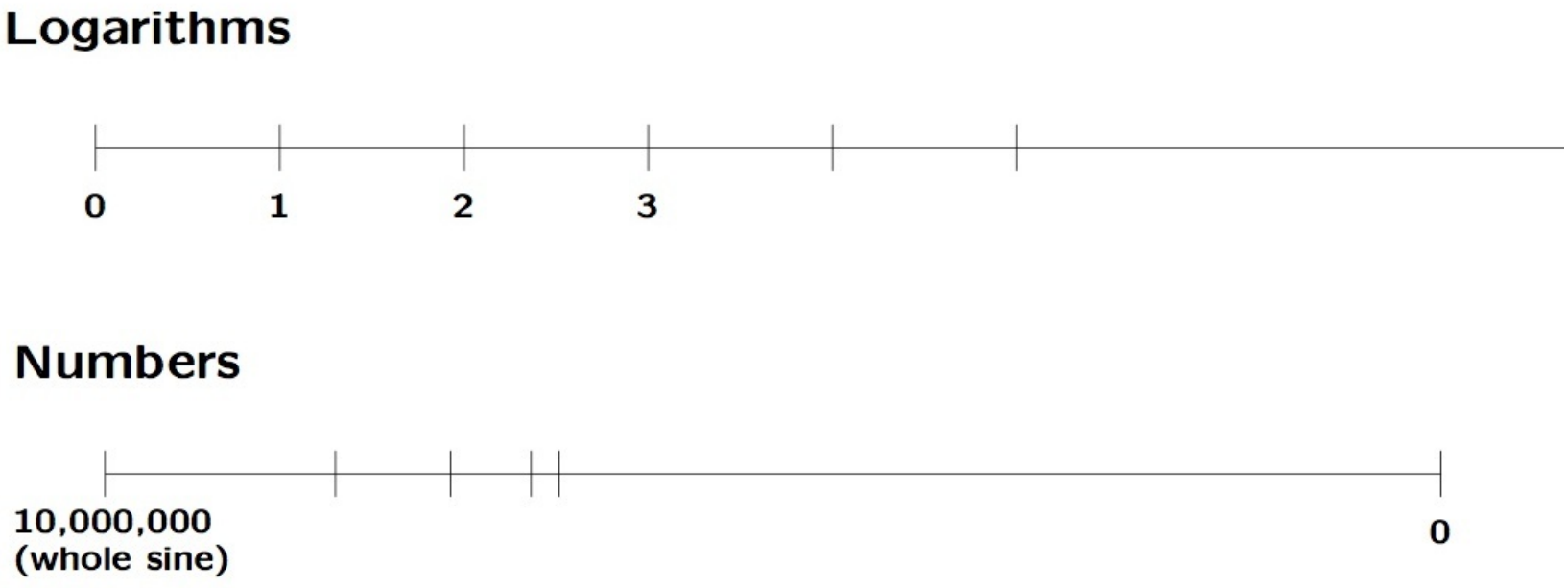


Sine of angle at centre varies between 0 and ± 1 as the labelled radius sweeps around the circle



Sine of angle at centre varies between 0 and $\pm 10,000,000$ as the labelled radius sweeps around the circle

Napier's logarithms



Napier's logarithms (1614)

In modern terms (i.e., not Napier's):

$$\text{if } y = 10^7\left(1 - 10^{-7}\right)^x, \text{ then } \operatorname{Nap\,log}\, y = x$$

$\operatorname{Nap\,log}\, 10^7 = 0$, $\operatorname{Nap\,log}\, 0$ is infinite, $\operatorname{Nap\,log}\, 1 = 161,180,956$

$$\operatorname{Nap\,log}\left(\frac{p \times q}{10^7}\right) = \operatorname{Nap\,log} p + \operatorname{Nap\,log} q$$

$$\operatorname{Nap\,log}\left(p \times q\right) = \operatorname{Nap\,log} p + \operatorname{Nap\,log} q - \operatorname{Nap\,log} 1$$

Note that $\operatorname{Nap\,log} x = 10^7 \ln\left(\frac{10^7}{x}\right)$

No notion of base, although Nap log 'nearly' has base $\frac{1}{e}$ — see Robin Wilson, *Euler's Pioneering Equation*, OUP, 2019, p. 101

Modifications by Napier and Briggs (1617)

Definition revised to remove the need to subtract $\mathrm{Nap}\log 1$

'Briggsian' logarithms have base 10 and $\mathrm{Log}\,1 = 0$, so that

$$\mathrm{Log}\,(p \times q) = \mathrm{Log}\,p + \mathrm{Log}\,q$$

Briggs produced *Logarithmorum chilias prima* (*The first thousand logarithms*) in 1617, followed by his *Arithmetica logarithmica* in 1624, which contained logarithms of 1 to 20,000 and 90,000 to 100,000, all to 14 decimal places (calculated by hand); the gap in the table was filled by Adriaan Vlacq in 1628

Napier's logarithms

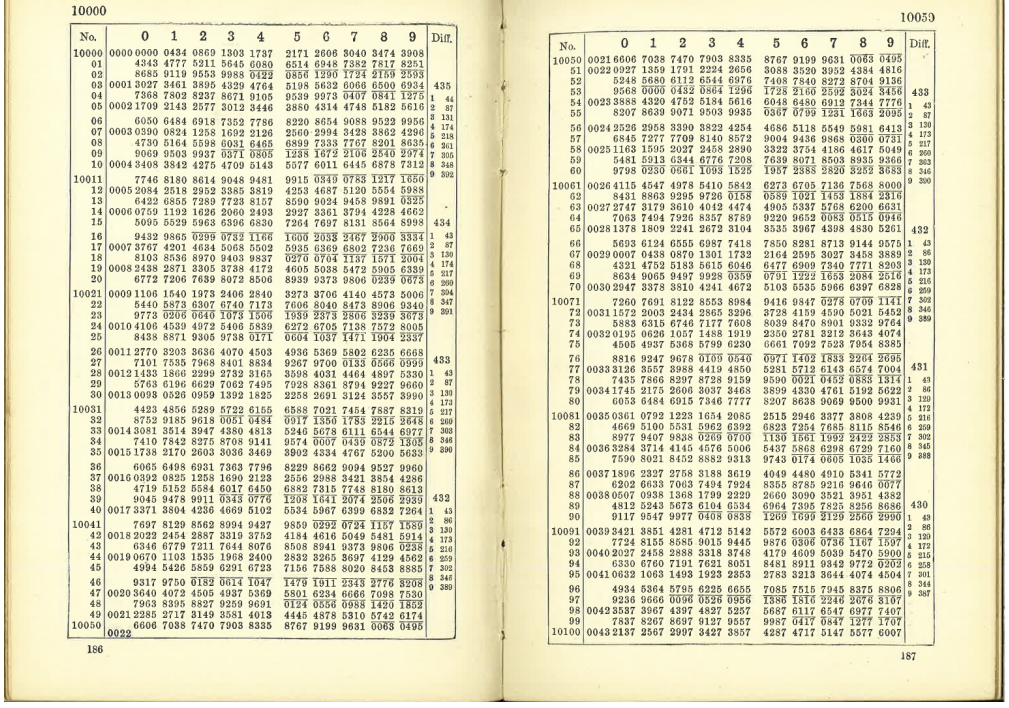
One last time:

> *The* **Logarithme** *therefore of any sine is a number very neerely expressing the line, which increased equally in the meane time, whiles the line of the whole sine decreased proportionally into that sine, both motions being equal-timed, and the beginning equally swift.*

Significance

Napier's logarithms:

- caught on very quickly
- a calculating aid (until the 1980s)
- logarithms rapidly came to have other interpretations (as you know, and as we shall see)


10000

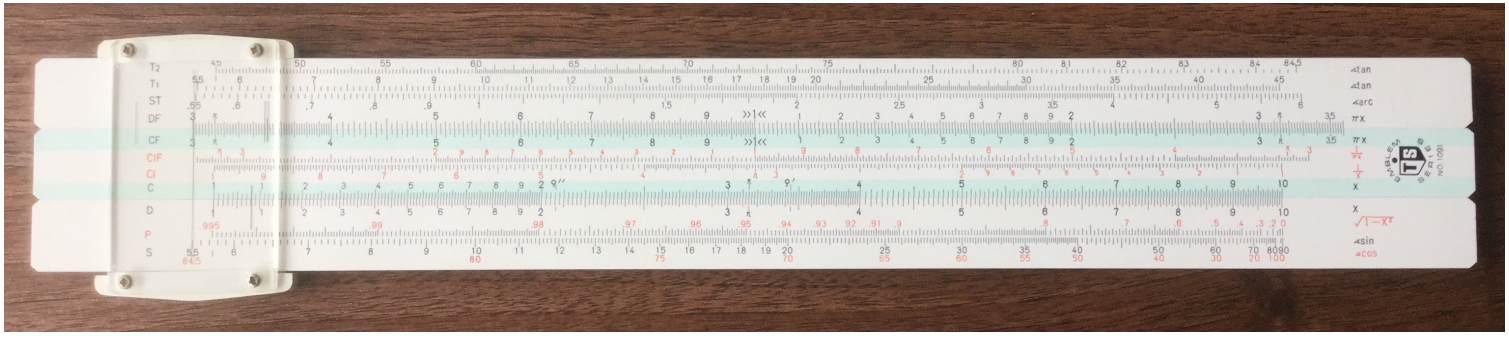
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08	4730	5164	5598	6031	6465	6899	7333	7767	8201	8635	6 261
09	9069	9503	9937	0371	0805	1238	1672	2106	2540	2974	7 305
10	0004 3408	3842	4275	4709	5143	5577	6011	6445	6878	7312	8 348
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14	0006 0759	1192	1626	2060	2493	2927	3361	3794	4228	4662	
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16	9432	9865	0299	0732	1166	1600	2033	2467	2900	3334	1 43
17	0007 3767	4201	4634	5068	5502	5935	6369	6802	7236	7669	2 87
18	8103	8536	8970	9403	9837	0270	0704	1137	1571	2004	3 130
19	0008 2438	2871	3305	3738	4172	4605	5038	5472	5905	6339	4 174
20	6772	7206	7639	8072	8506	8939	9373	9806	0239	0673	5 217
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22	5440	5873	6307	6740	7173	7606	8040	8473	8906	9340	8 347
23	9773	0206	0640	1073	1506	1939	2373	2806	3239	3673	9 391
24	0010 4106	4539	4972	5406	5839	6272	6705	7138	7572	8005	
25	8438	8871	9305	9738	0171	0604	1037	1471	1904	2337	
26	0011 2770	3203	3636	4070	4503	4936	5369	5802	6235	6668	
27	7101	7535	7968	8401	8834	9267	9700	0133	0566	0999	433
28	0012 1433	1866	2299	2732	3165	3598	4031	4464	4897	5330	1 43
29	5763	6196	6629	7062	7495	7928	8361	8794	9227	9660	2 87
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35	0015 1738	2170	2603	3036	3469	3902	4334	4767	5200	5633	9 390
36	6065	6498	6931	7363	7796	8229	8662	9094	9527	9960	
37	0016 0392	0825	1258	1690	2123	2556	2988	3421	3854	4286	
38	4719	5152	5584	6017	6450	6882	7315	7748	8180	8613	
39	9045	9478	9911	0343	0776	1208	1641	2074	2506	2939	432
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43	6346	6779	7211	7644	8076	8508	8941	9373	9806	0238	5 216
44	0019 0670	1103	1535	1968	2400	2832	3265	3697	4129	4562	6 259
45	4994	5426	5859	6291	6723	7156	7588	8020	8453	8885	7 302
											8 346
46	9317	9750	0182	0614	1047	1479	1911	2343	2776	3208	9 389
47	0020 3640	4072	4505	4937	5369	5801	6234	6666	7098	7530	
48	7963	8395	8827	9259	9691	0124	0556	0988	1420	1852	
49	0021 2285	2717	3149	3581	4013	4445	4878	5310	5742	6174	
10050	6606	7038	7470	7903	8335	8767	9199	9631	0063	0495	
	0022										

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10050

No.	0	1	2	3	4	5	6	7	8	9	Diff.
10050	0021 6606	7038	7470	7903	8335	8767	9199	9631	0063	0495	
51	0022 0927	1359	1791	2224	2656	3088	3520	3952	4384	4816	
52	5248	5680	6112	6544	6976	7408	7840	8272	8704	9136	
53	9568	0000	0432	0864	1296	1728	2160	2592	3024	3456	433
54	0023 3888	4320	4752	5184	5616	6048	6480	6912	7344	7776	1 43
55	8207	8639	9071	9503	9935	0367	0799	1231	1663	2095	2 87
											3 130
56	0024 2526	2958	3390	3822	4254	4686	5118	5549	5981	6413	4 173
57	6845	7277	7709	8140	8572	9004	9436	9868	0300	0731	5 217
58	0025 1163	1595	2027	2458	2890	3322	3754	4186	4617	5049	6 260
59	5481	5913	6344	6776	7208	7639	8071	8503	8935	9366	7 303
60	9798	0230	0661	1093	1525	1957	2388	2820	3252	3683	8 346
											9 390
10061	0026 4115	4547	4978	5410	5842	6273	6705	7136	7568	8000	
62	8431	8863	9295	9726	0158	0589	1021	1453	1884	2316	
63	0027 2747	3179	3610	4042	4474	4905	5337	5768	6200	6631	
64	7063	7494	7926	8357	8789	9220	9652	0083	0515	0946	
65	0028 1378	1809	2241	2672	3104	3535	3967	4398	4830	5261	432
66	5693	6124	6555	6987	7418	7850	8281	8713	9144	9575	1 43
67	0029 0007	0438	0870	1301	1732	2164	2595	3027	3458	3889	2 86
68	4321	4752	5183	5615	6046	6477	6909	7340	7771	8203	3 130
69	8634	9065	9497	9928	0359	0791	1222	1653	2084	2516	4 173
70	0030 2947	3378	3810	4241	4672	5103	5535	5966	6397	6828	5 216
											6 259
10071	7260	7691	8122	8553	8984	9416	9847	0278	0709	1141	7 302
72	0031 1572	2003	2434	2865	3296	3728	4159	4590	5021	5452	8 346
73	5883	6315	6746	7177	7608	8039	8470	8901	9332	9764	9 389
74	0032 0195	0626	1057	1488	1919	2350	2781	3213	3643	4074	
75	4505	4937	5368	5799	6230	6661	7092	7523	7954	8385	
76	8816	9247	9678	0109	0540	0971	1402	1833	2264	2695	
77	0033 3126	3557	3988	4419	4850	5281	5712	6143	6574	7004	431
78	7435	7866	8297	8728	9159	9590	0021	0452	0883	1314	1 43
79	0034 1745	2175	2606	3037	3468	3899	4330	4761	5192	5622	2 86
80	6053	6484	6915	7346	7777	8207	8638	9069	9500	9931	3 129
											4 172
10081	0035 0361	0792	1223	1654	2085	2515	2946	3377	3808	4239	5 216
82	4669	5100	5531	5962	6392	6823	7254	7685	8115	8546	6 259
83	8977	9407	9838	0269	0700	1130	1561	1992	2422	2853	7 302
84	0036 3284	3714	4145	4576	5006	5437	5868	6298	6729	7160	8 345
85	7590	8021	8452	8882	9313	9743	0174	0605	1035	1466	9 388
86	0037 1896	2327	2758	3188	3619	4049	4480	4910	5341	5772	
87	6202	6633	7063	7494	7924	8355	8785	9216	9646	0077	
88	0038 0507	0938	1368	1799	2229	2660	3090	3521	3951	4382	
89	4812	5243	5673	6104	6534	6964	7395	7825	8256	8686	430
90	9117	9547	9977	0408	0838	1269	1699	2129	2560	2990	1 43
											2 86
10091	0039 3421	3851	4281	4712	5142	5572	6003	6433	6864	7294	3 129
92	7724	8155	8585	9015	9445	9876	0306	0736	1167	1597	4 172
93	0040 2027	2458	2888	3318	3748	4179	4609	5039	5470	5900	5 215
94	6330	6760	7191	7621	8051	8481	8911	9342	9772	0202	6 258
95	0041 0632	1063	1493	1923	2353	2783	3213	3644	4074	4504	7 301
											8 344
96	4934	5364	5795	6225	6655	7085	7515	7945	8375	8806	9 387
97	9236	9666	0096	0526	0956	1386	1816	2246	2676	3107	
98	0042 3537	3967	4397	4827	5257	5687	6117	6547	6977	7407	
99	7837	8267	8697	9127	9557	9987	0417	0847	1277	1707	
10100	0043 2137	2567	2997	3427	3857	4287	4717	5147	5577	6007	

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Significance as a historical source

- Roles of translation in mathematics
- Concept of authorship in the 16th century
- Use of diagrams in mathematical texts
- Importance of informal/social communication, alongside published texts