# Mathematical Physiology

## Ian Griffiths

**Problem Sheets**

- Problem Sheet classes:
  - Tuesdays 2–3:30, Weeks 2, 4, 6, 8, Room C5.
    Tutor: Sam Palmer. TA: Siting Miao

  - Wednesday 11-12:30, Weeks 2, 4, 6, 8, Room C6.
    Tutor: Ian Griffiths. TA: Siting Miao

- Hand-in time for both classes: Friday 5pm of Weeks 1, 3, 5, 7.

- The first problem sheet will be a little shorter since you only have a week to attempt this.

## Special Topics

- For those who are attending and need to write a special topic on this course, I have uploaded a list of possible topics.

## Lectures

- The typeset lecture notes are detailed. But, everything you need will be covered in the lectures. (Sometimes I will point to the lecture notes for an additional proof.)

- The course is a little different to other mathematical courses. Here there will be just as much emphasis on coming up with the appropriate mathematical models as there is on solving them.

- To add a little interest and relevance, we will have some guest appearances from research experts in the field (in brain modelling, calcium dynamics,…).

- I will use a mixture of Powerpoint slides (which will be available on the course website) and writing on the whiteboard.

## Enzyme kinetics

- Enzymes are catalysts – they help convert other molecules (called substrates) into products but are not used up in the reaction themselves.

- Consider chemicals A and B reacting on collision to form chemical C with a rate k. (The rate depends on the molecule shapes and sizes and the temperature.)

## Transmembrane ion transport

- Cells are effectively bags of water.

- The water contains dissolved salts: NaCl and KCl, which dissolve into $Na^+$, $K^+$ and $Cl^-$ ions.

- They exist both inside and outside the cell, creating a potential difference.

- The cell walls are permeable – ions may be transported through the cell membrane, passing through pores called channels or gates.

- Osmosis is the mechanism by which water is transported across the cell membrane.

- Carrier mediated diffusion – a molecule hitches a lift by binding to a carrier molecule that is lipid soluble and can move through the membrane.

- Carrier mediated transport – a molecule binds to a protein that has an active site that may be exposed to the interior or exterior of the cell (e.g., glucose or aminio acid transport).

- Pumps – these exchange one ion for another, e.g., $Na^+$ for $K^+$ or $Na^+$ for $Ca^{2+}$.

# Summary of lecture 1

- The law of mass action:

$$\mathrm{A} + \mathrm{B} \xrightarrow{k} \mathrm{C} \qquad \frac{dC}{dt} = kAB$$

- Michaelis–Menten law:

$$c = \frac{s}{s + K'} \qquad \frac{ds}{dt'} = -\frac{\lambda s}{s + K'}$$

# PhD studentship on modelling and data analysis of in-vitro fertilization

- Based at Cardiff Mathematics and supervised by Dr Katerina Kaouri (lead supervisor), Dr Thomas Woolley, Prof Karl Swann (Cardiff Biosciences), Prof Krasimira Tsaneva-Atanaseva (Exeter) and Dr Cameron Hall (Bristol).

- The student should have a strong quantitative background (degree in Mathematics, Physics, Engineering or Computer Science).

- Open to both home and international students.

- The funding is for four years.

- Deadline: **2nd November**. Start date: October 2023

- More info and how to apply: www.findaphd.com/phds/project/calcium-signalling-in-in-vitro-fertilization-developing-a-non-invasive-diagnostic-tool/?p147305

- Contact: kaourik@cardiff.ac.uk

## Transmembrane ion transport

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# A model for carrier mediated transport

- $C_i$ = a state with a binding site exposed to the interior.
- $C_e$ = a state with a binding site exposed to the exterior.
- $C_e$ can bind with a substrate molecule in the exterior $S_e$ to make a product $P_e$.
- $C_i$ can bind with a substrate molecule in the interior Si to make a product $P_i$ (with the same rates as the exterior).
- $P_i$ can turn into $P_e$ . This is the carrier doing its ‘rotation’.
- $C_i$ can turn into $C_e$. This is the carrier site rotating without any substrate on it. We assume this occurs at the same rate as the rotation with substrate on it.

## Summary of lecture 2

- We can write down an ODE model for carrier-mediated transport. The model tells us the rate at which ions can be transferred from the outside of the membrane to the inside in steady state.

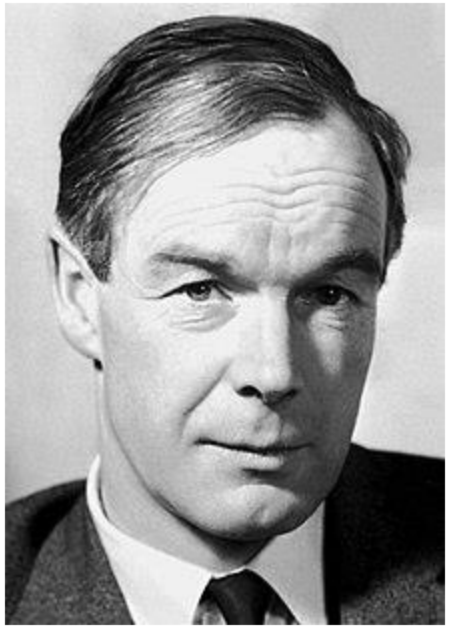
- Active transport involves moving molecules against concentration gradients. This requires energy.

- Ions will move across a membrane wall to balance concentration. However, this might lead to a difference in charge. The system reaches an equilibrium when the diffusive flux balances the ionic flux.

- This occurs at the Nernst potential:

$$V_S = \frac{RT}{zF} \ln\left(\frac{c_e}{c_i}\right)$$

# The Hodgkin–Huxley model



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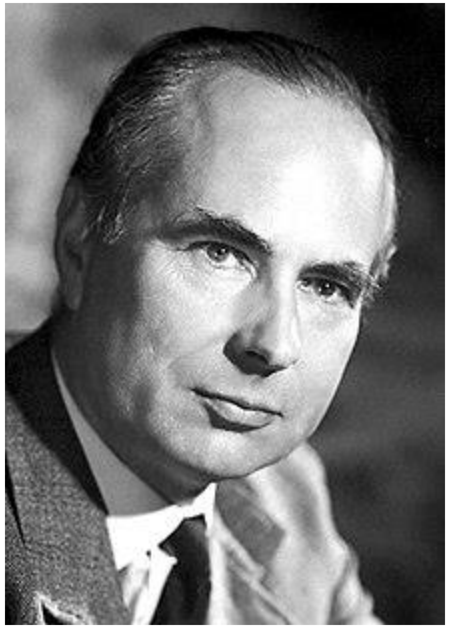

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## The Hodgkin–Huxley model

- The nervous system is a communication systems formed by nerve cells or neurons.
- Information is propagated along long cylindrical segments called axons by electrochemical signals.
- Communication between cells occurs at junctions between synapses to the dendrites.
- If a small current is applied for a short time the membrane potential simply returns to its resting potential when the current is removes.
- But for a sufficiently high current, the membrane potential undergoes a large excursion – an action potential – before returning to its resting value.
- Signals are transmitted by the propagation of these action potentials.