Matrix eigenvalue problems and linear systems

Introduction

Eigenvalue problems $Ax = \lambda x$ and linear systems $Ax = b$ are the two central problems addressed in numerical linear algebra. These are both of fundamental importance in scientific computing, as most problems eventually boil down to one of the two (or their variant), taking up a bulk of the computational effort.

The Singular Value Decomposition (SVD) $A = U\Sigma V^T$, which is closely related to eigenvalue problems, is also worth the special mention for its enormous importance in applications in data science.

Project

While extensive studies have been devoted to these problems, a number of open problems remain, and many interesting questions have been identified with the rapid surge of data science, with significant ramifications in applications. For example, topics recently developed and awaiting further investigations include QR factorisation with column pivoting for low-rank approximation, the randomised Kaczmarz (and related coordinate descent) method for linear systems, extraction of leading eigenvalues or singular vectors, and genearlisations of Sylvester's law of inertia. There is also room for further explorations in classical topics in matrix analysis and eigenvalue perturbation theory.

This project aims to explore, experiment, examine and potentially resolve problems in numerical linear algebra.

Prerequisites

Prelims Linear Algebra. Part A Numerical Analysis is highly recommended. Part A courses on Probability and Complex Analysis would also be helpful.

Reading

The general recommended textbook is: L. N. Trefethen and D. Bau, Numerical Linear Algebra, SIAM, 1997.

For specific topics, a subset of these references would be of interest.

R. M. Gower and P. Richtarik. Randomized iterative methods for linear systems. SIAM J. Matrix Anal. Appl., 36(4):1660-1690, 2015.

M. Gu and S. C. Eisenstat. Efficient algorithms for computing a strong rank-revealing QR factorization. SIAM J. Sci. Comp, 17(4):848-869, 1996.

Y. Nakatsukasa. Sharp error bounds for ritz vectors and approximate singular vectors. Math. Comp., 89(324):1843–1866, 2020.

G. W. Stewart and J.-G. Sun. Matrix Perturbation Theory. Academic Press, 1990.

T. Strohmer and R. Vershynin. A randomized Kaczmarz algorithm with exponential convergence. J. Fourier Anal. Appl., 15(2):262, 2009.