ORTHOGONAL POLYNOMIALS

Introduction

Orthogonal polynomials are crucial in several areas, such as numerical analysis and physics. In numerical analysis, the power basis is notoriously ill-conditioned. High order methods require certain orthogonality, which leads to good condition numbers and sparsity. In quantum mechanics, orthogonal polynomials or special functions provide analytic solutions for some important systems.

Although orthogonal polynomial is a classical subject, current research (e.g., on the finite element methods) raises several questions which are not fully solved in the literature. For example, orthogonal polynomials on general cell shapes (e.g., simplicial and prismatic cells), optimising condition number in different norms, fast evaluation, vector and matrix-valued problems.

The project

The starting point will be implementing some standard univariate orthogonal polynomials and comparing the condition number and sparsity in various norms. After that, we can go into several possible directions: simplicial and prismatic cells, Bernstein basis, vector/matrix-valued problems.

Prerequisites

There are no prerequisites. Familiarity with notions of norms, inner products and orthogonality from linear algebra would be helpful.

Reading

An encyclopaedia about multivariate orthogonal polynomials is

- Orthogonal polynomials of several variables (second edition); C. Dunkl, Y. Xu, Cambridge University Press.

Multi-variate Jacobi polynomials on simplices with numerical experiments can be found in

- Well-conditioned Orthonormal Hierarchical $\mathscr{L}_2$ Bases on $\mathbb{R}^n$ Simplicial Elements. J. Xin and W. Cai, Journal of scientific computing, 50(2), pp.446-461 (2012).

Discussions on univariate and multivariate polynomials in the context of finite element methods can be found in some chapters of the thesis

- High Order Finite Element Methods for Electromagnetic Field Computation; S. Zaglmayr, PhD thesis, JKU Linz.