Computational Mathematics - Problem Sheet 4

MT 2022

Once you have completed the exercises, use the `publish` *command to generate a .pdf file of your solutions.*

The famous sequence of **Fibonacci numbers** F_i is constructed by combining the initial conditions $F_0 = 1$ and $F_1 = 1$ with the recursive equation

$$F_i = F_{i-1} + F_{i-2},$$

so the first few terms are $1, 1, 2, 3, 5, 8, 13, \ldots$ The limit of successive ratios F_i / F_{i-1} exists and is called the **Golden ratio**:

$$\phi = \lim_{i \to \infty} \frac{F_i}{F_{i-1}}.$$

Its actual value is $\phi = \frac{1+\sqrt{5}}{2}$.

1. Write a function that generates the first n Fibonacci numbers.
2. Using your function from Q1, write a function that outputs the n-th approximation (i.e., F_{n+1}/F_n) to the Golden Ratio.
3. Use the function from Q2 to compute an approximation to the Golden Ratio with accuracy 10^{-7}. Give your answer as a fraction.
4. Plot a graph of the error of your approximations against n. Your n values should range from $n = 1$ to the value corresponding to the error determined in Q3.