(6

3.

$$h_t = \frac{1}{3}(h^3 h_x)_x \quad , \quad \int h\,dx = 1$$

Similarity solution $h = \frac{1}{t^\alpha} f\left(\frac{x}{\mu t^\alpha}\right)$

write $\eta = \frac{x}{\mu t^\alpha}$

$$\Rightarrow \quad -\frac{\alpha}{t^{1+\alpha}} f - \frac{\alpha}{t^{1+\alpha}} \eta f' = \frac{1}{12} \frac{1}{t^{4\alpha}} \frac{1}{\mu^2 t^{2\alpha}} (f^4)''$$

$$\& \int_{-\infty}^{\infty} \mu h \, d\eta = 1$$

Choose $1+\alpha = 6\alpha \quad \Rightarrow \quad \underline{\alpha = \frac{1}{5}}$

then $-\frac{1}{5}(\eta f)' = \frac{1}{12\mu^2}(f^4)''$

$$\Rightarrow \quad (f^4)' = -\frac{12\mu^2}{5}\eta f$$

$$\frac{(f^4)'}{(f^4)^{1/4}} = -\frac{12\mu^2}{5}\eta$$

$$\frac{4}{3}(f^4)^{3/4} = -\frac{6\mu^2}{5}(\eta^2 - \eta_0^2)$$ where $f=0$ at $\eta = \eta_0$ ($f \equiv 0$ for $\eta > \eta_0$)

$$f = \left[\frac{9}{10}\mu^2(\eta_0^2 - \eta^2)\right]^{1/3}$$

Select μ as convenient, e.g. s.t $\eta_0 = 1$

then $f = \left(\frac{9}{10}\mu^2(1-\eta^2)\right)^{1/3}$

and $\int_0^1 \mu \left[\frac{9}{10}\mu^2(1-\eta^2)\right]^{1/3} d\eta = 1 \quad \Rightarrow \quad \mu = \frac{1}{\left[\int_0^1 \left\{\frac{9}{10}(1-\eta^2)\right\}^{1/3} d\eta\right]^{3/5}}$