How to Write an [M] Special Topic

M.Sc. in Mathematical Modelling & Scientific Computing,
Additional Skills

2nd November 2022

Key Points

- go beyond the lectures, either by looking at more advanced mathematical models or investigating given models in more detail;
- additional details should be provided over and above what is given in the source materials. For example, extra steps in the calculations could be given, alternative methods could be used, or there could be an alternative derivation of a model along with a critical discussion of the modelling ideas;
- demonstrate that a range of sources has been consulted (and these should be referenced appropriately). A suitable critical literature review would be an appropriate form for a special topic report;
- summarise work at end and cite sources throughout.

Example: Further Mathematical Biology

One of the topics in the Further Mathematical Biology course concerns trans-membrane ion transport which is modelled using the Hodgkin-Huxley and Fitzhugh-Nagumo equations.

Idea for a project is to investigate these models further, giving a derivation of the model, perform sensitivity analysis of the parameters and investigate the emergence of periodic solutions.

Title Page

The title page should consist of the title of the special topic, the name of the lecture course and your candidate number (NOT your name). You may also add a university crest. If this is all that is on the title page it does not contribute to the page count. If you add a table of contents or abstract then it does contribute to the page count. (Neither a table of contents nor an abstract is necessary.)

Title Page

Title of Special Topic

Lecture Course

Candidate Number: xxxxxx

Introduction

Give an introduction to the problem — what are you aiming to model, maybe a very brief history of the model and any relevant biology.

Derivation of the Model

Of course the model already exists so it is likely that your derivation will follow a book or paper — be sure to reference this. Then give a derivation of the model in your own words, making sure you introduce all notation as you use it and fill in any gaps in the reference(s) you are following.

If the derivation is long and there are several equations, it can be helpful to summarise them at the end of this section. Make sure you also give any appropriate initial and boundary conditions so that you have a complete model.

Non-dimensionalisation

Non-dimensionalise the equations next, or give a reason why you don't! Non-dimensionalisation may help to identify different time-scales in a problem or help to see that some terms are very small and can perhaps be neglected.

Hodgkin-Huxley Problem

The problem to be studied is

$$\begin{aligned} C\frac{\mathrm{d}v}{\mathrm{d}t} &= I_{\mathrm{app}} - \bar{g}_K n^4 (v - v_K) - \bar{g}_{NA} m^3 h (v - v_{Na}) - \bar{g}_I (v - vl) \\ \frac{\mathrm{d}n}{\mathrm{d}t} &= \alpha_n(v)(1-n) - \beta_n(v) n \\ \frac{\mathrm{d}m}{\mathrm{d}t} &= \alpha_m(v)(1-m) - \beta_m(v) m \\ \frac{\mathrm{d}h}{\mathrm{d}t} &= \alpha_h(v)(1-h) - \beta_h(v) h \end{aligned}$$

where there are lots of functions to be defined, and varying $I_{\mathrm{app}}(t)$ is of interest.

A range of different works consider different descriptions for the function $I_{\mathrm{app}}(t)$ and investigating some of these makes a nice special topic.

Example 1

Can set I_{app} to be zero except in a small time interval when it is a fixed positive constant. Then a numerical solution can be calculated. It is possible to experiment with the length of the time interval for which I_{app} is non-zero and with the size of the non-zero term. A numerical solution in Matlab would be possible then it is easy to adjust parameters. What effect do they have and how do we interpret the results biologically?

(In [M] special topics you should always be trying to relate the mathematical results to the underlying problem.)

Example 2

For the Hodgkin-Huxley model a phase plane analysis can be performed. Earlier analysis would have shown two time scales to the problem and while v and m vary rapidly, n and h are approximately constant on the fast time scale, thus the (v, m) phase plane can be considered (with $I_{\text{app}} = 0$). This has been done in the literature so be sure to cite references clearly and to fill in any missing details.

The behaviour of the solution will depend on parameters in the model — what are the effects of these parameters and how can we interpret the results biologically?

Example 3

An alternative is to let I_{app} be constant. The size of this constant has an effect on the resulting solution. This can be investigated using a linear stability analysis and the results of this analysis can be compared with numerical simulations.

Conclusion

Make sure you write a summary of the project at the end, perhaps giving a critical discussion of the model(s) presented and suggestions for further investigation.