# BO1.1. History of Mathematics

## Sheet 0 — HT23

## Reading Course: Christmas Vacation Reading

# Outline

The main subject of this year's reading course is one of Isaac Newton's lesser-known texts, his *Arithmetica universalis*, first published in Latin in 1707. Based on lectures that Newton gave as Lucasian Professor in Cambridge, the *Arithmetica universalis* is an introductory treatise on arithmetic, and on the algebraic and numerical solution of equations. Though not as influential as Newton's other writings, it contains much that is of interest for the study of algebra in the early eighteenth century. Recall from the lecture course that many commentators have dismissed the period between Cardano and Lagrange (the mid-sixteenth to late eighteenth centuries) as a time of inactivity in the study of algebra. We saw, however, that although there were arguably no major advances during these years, mathematicians were nevertheless very active in further unpicking the structure of polynomial equations and the nature of their roots, developing new ways of manipulating equations in order to make them more amenable to solution, and providing methods for the numerical solution of equations where algebraic methods failed. Newton's text was very much a part of this process, and contains ideas that are still taught today. The reading course will consist of three parts:

1. the detailed reading and analysis of Newton's methods;
2. the examination of commentaries on Newton's work by British authors Colin Maclaurin and Nicholas Saunderson;
3. the investigation of how Newton's ideas were taken up by continental writers, in particular Leonhard Euler and Joseph-Louis Lagrange.

# Main texts

Our first main text (which you will need for the vacation reading) is the 1728 revised English translation of the *Arithmetica universalis*:

- Isaac Newton, *Universal arithmetick, or A treatise of arithmetical composition and resolution*, London, 1728.

This can easily be found in an electronic form via SOLO, as can two other of our main texts:

- Colin Maclaurin, *A treatise of algebra*, London, 1748;
- Nicholas Saunderson, *The elements of algebra*, Cambridge, 1740.


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For the Euler materials, you will need to visit the Euler Archive (eulerarchive.maa.org) in order to find the following papers and their English translations:

- Leonhard Euler, 'Demonstratio gemina theorematis Neutoniani, quo traditur relatio inter coefficientes cuiusvis aequationis algebraicae et summas potestatum radicum eiusdem' ('A double demonstration of a theorem of Newton, which gives a relation between the coefficient of an algebraic equation and the sums of the powers of its roots'), *Opuscula varii argumenti*, vol. 2 (1746–51), pp. 108–120; also published in: *Opera omnia*, series 1, vol. 6, pp. 20–30; E153.[1]

- Leonhard Euler, 'Nouvelle méthode d'éliminer les quantités inconnues des équations', *Mémoires de l'académie des sciences de Berlin*, vol. 20 (1764), pp. 91–104; also published in: *Opera omnia*, series 1, vol. 6, pp. 197–211; E310.

Finally, the easiest way to find the following paper by Lagrange is to look in his collected works (*Oeuvres de Lagrange*) on the website Gallica (gallica.bnf.fr), the digital library of the Bibliothèque nationale de France:

- Joseph-Louis Lagrange, 'Sur la résolution des équations numériques', *Mémoires de l'Académie des Sciences de Berlin*, vol. 23 (1767), pp. 311–352; also published in: *Oeuvres de Lagrange*, vol. 2, pp. 539–578.

English translations of selected parts of this paper will be provided. Other reading materials will be recommended during HT.

# Vacation reading

In order to get started, please read the following parts of Newton's *Universal arithmetick* over the vacation, and be prepared to discuss them in the class in HT week 1:[2]

- pp. 1–28 and 33.2–37 (on notation, operations, fractions, roots, reduction);

- pp. 38–41.1 and 45.2–47 (on divisors);

- pp. 51–52 (on radicals);

[1] In the early twentieth century, the Swedish mathematician Gustaf Eneström compiled a comprehensive list of Euler's publications and assigned them index numbers, now referred to as 'Eneström numbers' and denoted 'Exxx'.

[2] In what follows, the notation 'p. $N.m$' means 'page $N$, paragraph $m$'.


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- pp. 55–67.1 (on equations).

You will find much of this reading to be mathematically easy, but you should familiarise yourself with Newton's notation and vocabulary, just as his original readers were expected to do. Note also that some pages has been omitted from the reading to save length and repetition, but you should always check briefly what has been missed. When reading this material, try to keep the following questions in mind, as these will form the basis for the discussion in the class:

- How easy or difficult do you consider the text to be?
- How even or graded is the difficulty?
- Does Newton provide sufficient explanation?
- Why does the square root method work (p. 28)?
- How does the 'divisor' method work (p. 38)?
- Why is Newton interested in roots of radicals (p. 51)?


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