BO1 History of Mathematics Lecture XVI Concluding miscellany

MT 2022 Week 8

Summary

- ► The exam (briefly)
- Points to ponder
- ► The history of the history of mathematics*
- ► Hilary Term reading course

Structure of the exam paper

Section A

- Six extracts given
- Choose two and comment on the context, content, and significance
- Each extract is worth 25 marks
- Each extract is typically one short paragraph it will relate to a topic that we have studied, though you may not have seen the precise extract before
- By way of practice, choose any quotation or short extract that has appeared on the lecture slides

Section B

- Three essay topics given
- Choose one
- Answer worth 50 marks

Typical exam questions (Section B)

Q. Discuss, with reference to specific examples, how concept X (or terminology Y, or notation Z, ...) has developed between 1600 and 1900.

Q. Discuss with reference to specific examples, how attitudes towards X have changed between 1600 and 1900.

Q. Discuss the significance of text X.

Q. Describe some aspects of the work of major figure X.

Points to ponder (1)

What is the history of mathematics?
What does it mean to study the history of mathematics?
What is mathematics?

Points to ponder (2)

What do you think the words 'mathematics' and 'mathematician' have meant throughout this course?

Have they had the same meanings throughout?

More generally, have they had the same meanings throughout history?

Points to ponder (3)

If we choose to understand the word 'mathematics' differently, how does this change our view of the history of mathematics?

How could a revised definition of 'mathematics' change the selection of people and cultures who appear in the story?

What does the study of the history of mathematics have to tell us about the way in which we approach mathematics nowadays?

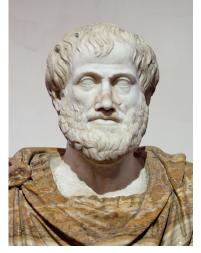
Historiography of mathematics

According to the *OED*:

historiography, n.

- 1. The writing of history; written history.
- 2. The study of history-writing, esp. as an academic discipline.

Ancient histories of mathematics

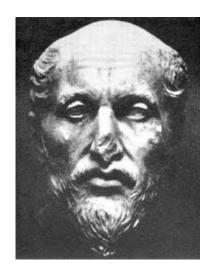


Aristotle (384–322 BC)

Eudemus (4th century BC)

- Student and editor of Aristotle
- ► History of Arithmetic
- History of Geometry
- History of Astronomy

Biographical background



Proclus's commentary on Euclid's *Elements* (5th century AD)

- (Spurious?) biographical details
- Built on anecdotes provided by Pappus (4th century AD)

Later historical attributions



a full understanding of geometry "requireth diligent studie and reading of olde auncient authors"

Renaissance humanist attitudes

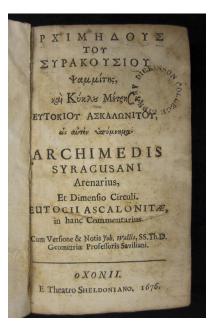


Sir Henry Savile (1549–1622)

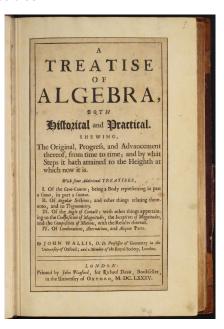
The teaching of mathematics should be founded on humanist principles:

- it should stem from the works of classical antiquity;
- scholars ought to have a concern for the history of their subject;
- they should actively seek to restore and edit surviving texts.

Renaissance humanist attitudes



Nationalist attitudes



Comprehensive histories of mathematics

171

HISTOIRE

DES

MATHEMATIQUES,

DANS laquelle on rend compte de leurs progrès de puis leur origine jusqu'à nos jours; où l'on expofe le tableau & le dévelopement des principales découvertes, les contestations qu'elles ont fait naître, & les principaux traits de la vie des Mathématiciens les plus célebres.

Par M. MONTUCLA, de l'Académie Royale des Sciences & Belles-Leures de Prusse.

Multi pertransibunt & augebitur scientia. Băcon-

TOME PREMIER.



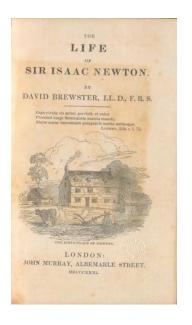
A PARIS,

Chez CH. ANT. JOMBERT, Imprimeur-Libraire du Roi pour l'Artillerie & le Génie, rue Dauphine, à l'Image Notre-Dame,

M. D C C. L V I I I.

Avec Approbation & Privilege du Ro

Lauding the great mathematicians

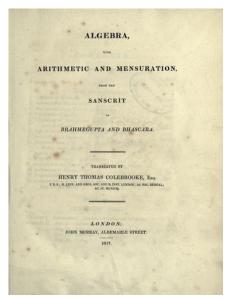


Adding greater nuance



See: Adrian Rice, 'Augustus De Morgan: historian of science', *History of Science* 34 (1996), 201–240

Awareness of mathematics beyond Europe



See: Ivahn Smadja, 'Sanskrit versus Greek 'proofs': history of mathematics at the crossroads of philology and mathematics in nineteenth-century Germany', Revue d'histoire des mathématiques 21(2) (2015) 217–349

Anecdotal history



Who studies the history of mathematics?

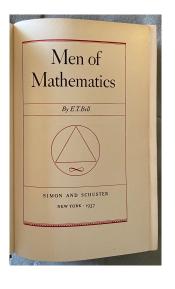
- Mathematicians?
 - Because mathematical knowledge is key?
 - Because only mathematicians will read it?
 - Suitable retirement project?
- ► Historians?
 - Because only an understanding of historical context makes it history?
 - ► For better integration into wider historical scholarship?
- Scholars who are somewhere in between?

Professionalisation



Otto Neugebauer (1899–1990)

Popularisation



- Humanisation of mathematics
- Mathematical myth-making
- Mathematical anecdotes for community-building
- Drawback: reinforcement of a particular view of the subject

until you obtain 2. It was therefore indeed a new idea to duplicate it by dividing 2 by n.

We do not know in whose brain this thought arose for the first time, nor when this happened. It certainly occurred long before the era of our texts, for the (2 : n)-table of the Rhind papyrus, which includes all the odd numbers from n = 3 to n = 101, was not constructed all at one time; its separate parts were computed by different methods. The oldest section contains the denominators which are divisible by 3; without exception, they all proceed according to the same rule:

$$2: 9 = \overline{6} + \overline{18}$$

 $2: 15 = \overline{10} + \overline{30}$
 $2: 21 = \overline{14} + \overline{42}$

In these cases the division 2 : 3k is simply a confirmation of a known result. In the other cases (certainly from n = 11 on), the duplication appears to have been obtained by actually carrying out the division of 2 by s. The text exhibits the divisions more or less explicitly, as in the following examples (2:5 and 2:7)

What part is 2 of 5? 3 is 1 + 3. 15 is 3.

i.e. a third of 5 is 1 + 3, a fifteenth is 3; these add up to 2. The result of the division is therefore 3 + 15; the terms 3 and 15 are clearly visible because they are written in red. In our "translation" the red symbols have been printed in bold-face type

In this manner the work proceeds. In dividing 2 by 5, 9, 11, 17, 23, 29 and a few of the larger integers, the 3 sequence is used, i.e. the sequence of fractions 3, 3, 6, 12, ...; but the division by 7 and 13 employs only the 2-sequence (2, 4, 8, . .). It turns out that only in these two cases the 2-sequence produces a simpler result than the 3-sequence. For instance, the use of the 2-sequence would, in calculating 2:11, lead to the result 2:11=8+22+88, while the 3sequence gives $2:11=\overline{6}+\overline{66}$, which, having fewer terms and smaller de-

nominators, is obviously to be preferred

The calculations which have been reproduced here certainly tell their own story. In the case 2:7, the number 4, placed in front of 28, indicates where 28 comes from, viz. from 4 x 7, the further details being shown in an auxiliary column. THE EGYPTIANS

The results of the divisions 2: n are summarized in the following table, which does not include divisors that are divisible by 3, all of which follow the rule $2:3k=2k+\overline{6k}$

```
2:5-3+15
                                                         2: 53 = 30 + 318 + 795
2: 7 - 4 + 28
                                                         2: 55 = 30 ± 330
2:11 = 6 + 66
                                                         2: 59 = 36 + 236 + 531
2:13 = 8 + 52 + 104
                                                         2: 61 - 40 + 244 + 488 + 510
2:17 = \overline{12} + \overline{51} + \overline{68}
                                                         2: 65 - 39 + 195
2:19 = \overline{12} + \overline{76} + \overline{114}
                                                         2: 67 = 40 + 335 + 536
2:23 = \overline{12} + \overline{276}
                                                         2: 71 = 40 + 568 + 710
2:25 = \overline{15} + 75
                                                         2: 73 = \overline{60} + \overline{219} + \overline{292} + \overline{365}
2:29 = \overline{24} + \overline{58} + \overline{174} + \overline{232}
                                                         2: 77 = \overline{44} + \overline{308}
                                                         2: 79 = \overline{60} + \overline{237} + \overline{316} + \overline{790}
2:31 = \overline{20} + \overline{124} + \overline{155}
                                                         2: 83 = 60 + 332 + 415 + 498
2:35 = 30 + 42
                                                         2:85 = \overline{51} + \overline{255}
2:37 = \overline{24} + \overline{111} + \overline{295}
                                                         2:89 = \overline{60} + \overline{356} + \overline{534} + \overline{890}
2:41=\overline{24}+\overline{246}+\overline{328}
                                                         2: 91 = 70 ± 130
2:43 = \overline{42} + \overline{86} + \overline{129} + \overline{301}
                                                         2:95 = \overline{60} + \overline{380} + \overline{570}
2:47=\overline{30}+\overline{141}+\overline{470}
                                                         2: 97 = 56 + 679 + 776
2:49 = \overline{28} + \overline{196}
2:51 = 34 + 102
                                                        2:101 - \overline{101} + \overline{202} + \overline{303} + \overline{605}
```

Beginning with 2:31, the form of presentation changes; the calculations are given in abbreviated form. But, what is more important, the method of calculation changes; another idea is introduced. While up to this point, all divisions were carried out by means of the 2-sequence and the 3-sequence, the divisions 2:31 and 2:35 proceeded quite differently, as is seen from the following examples:

What part is 2 of 31? $\overline{20}$ is $1 + \overline{2} + \overline{20}$, $\overline{124}$ is $\overline{4}$, $\overline{155}$ is $\overline{5}$ Computation:

The start of the computation of 2:31 is easy to account for, since division of 31 by 10, and halving of the result shows that $\frac{1}{20}$ of 31 is $1 + \overline{2} + \overline{20}$. This fraction is to be increased so as to produce 2. How did the calculator hit upon the idea that this requires 4 + 5? It checks; for the leather scroll has the relation

Rewriting the history of mathematics

On the Need to Rewrite the History of Greek Mathematics

SABETAI UNGURU Communicated by W. HARTNER

'History is the most fundamental science, for there is no human knowledge which cannot loss its scientific character when men forget the conditions under which it originated, the questions which it answered, and the function it was created to serve, A great part of the mysticism and superstition of cluented men consists of knowledge which has broken loose from its historical moorings.'

BRALLINE EXPRESSIVES.

"It would not occur to the modern mathematician, who uses algebraic symbols, that one type of geometrical progression [Le., 1, 2, 4, 8] could be more perfect better deserving of the name than another. For this reason algebraic symbols should not be employed in interpreting such a passage as ours [Le, Plato, Thanacas, 32A, B, Thanacas,

'Any historian of mathematics conscious of the perils and pitfalls of Whig history quickly discovers that the translation of past mathematics into modern symbolism and terminology represents the greatest danger of all. The symbols and terms of a modern mathematics are the bearers of its concepts and methods. Their application to historical material always involves the risk of imposing on that material, a content it does not in fact possess.'

The previous string of quotations is (most certainly) not illustrative of the ways in which the history of mathematics has traditionally been written. The authors of the quotations themselves have not always practiced what they occa-

Greek Science Its Meaning For Ux (Harmondsworth: Penguin Books, 1953), 311.
 Plato's Cosmology (New York: The Liberal Arts Press, 1957) 49.

Sabetai Unguru, 'On the need to rewrite the history of Greek mathematics', *Archive for History of Exact Sciences* 15 (1975), 67–114

⁵ The Mathematical Curves of Pierre de Fermal (1601–1665) (Princeton, N.J.: Princeton University Press, 1973), XII–XIII.

A broader perspective

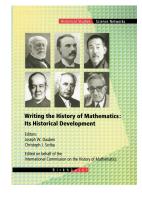


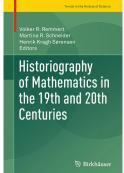
A broader perspective

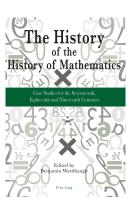


Brigitte Stenhouse, *Mary Somerville: Being and Becoming a Mathematician*, PhD thesis,
The Open University, 2021

Historiography of mathematics: references







How do we know what we know about ancient Egyptian mathematics?

3 Chapter 1 Ancient Mathematics

numbers developed to the point where the same digit it represented 00 as well. We do not have why the Ballysheims decided not have one large unit represent 60 small units and then adapt this method for their numeration systems. One conjecture is that 60 is creatly divisible by many small integers. Therefore, functional value of the "large" unit could easily divisible the small integers. Therefore, functional value of the "large" unit could easily appear in the small integers. Therefore, functional value of the "large" unit could easily appear in sill in size in our unit for rapid and time measurement units preserved over the central in astronomical contexts and only an irreplaced large not sweetly contents.

There is no rooted of the written unabor system of nozion India, but there is lineary offence that muscles system of an object held only offence about the index orange acceptance and explored system in the object of written numbers are available. Originally, the system was mixed. There was explored system inside to the lennic wite superact symbols of the numbers it through 9 and 10 through 90. For larger numbers, the system was a multiplicative one similar to Chinece. For example, the system was a multiplicative one similar to the Chinece. For example, the system was a multiplicative one similar to the Chinece and the symbol for 2 and 4 through 90. For larger mixed, the system of the system of

1.3 ARITHMETIC COMPUTATIONS

Doe the types of writing numbers came into axistance, all of the cilitations using changes and consideration of the three states attentive general-anditions, subspection, multi-operating with fractions. These rules may be considered as some of the earliest appointing and another three control and operating with fractions. These rules may be considered as some or to be entired appointing and another three controls are the entired appointing to the entired appointing and another control and appoints are some to a some or to a different produces, to fact, motions undemantics on the characterized an appointing on the control and appoints are also considered from the control and appoints are also considered and another applies or implicit, to exist the solutions are to the control and a tool and there proceed to see an algorithm, where also considered appoints was decorred why it of the algorithm, with the control and a tool and the proceeds to see an algorithm, which is increasingly complete states are all appoints where all appoints was decorred with put of the algorithm, we will decorred to a possible entire and appoints where all appoints was decorred. And the allowed the problem of the algorithm was decorred to problem the problem of each of the algorithm was decorred as a possible and an allowed the problem of the algorithm was decorred to a possible and an allowed the problem of the allowed to a possible and an allowed the problem of the allowed to a possible and allowed th

their andems who alsold the external question "buff", in the Egyptian Interpolytic grouping years, addition is simple enought. Combine the units, then the beats due to the Egyptian Interpolytic grouping years, and the external properties of the external properties

1.3 Arithmetic Computations

Such a simple algorithm for addition and subtraction is not possible in the hierarch system. For these operations, the malternatical paper; in one provide much reinforce; the answers to addition and subtraction problems are merely written down. Most probably, the surbushed addition tables. At some point these would have existed in viriation form, but a composet sorthe would, of course, have memorized them. The sorthest presumably used the addition tables in overner for subtraction problems.

The Egyptian algorithm for multiplication was based on a continual doubling process. For multiply two numbers as and, bit each would find not the own the pits. In Brownidd then double each number in the pair repeatedly, until the next doubling would cause the first element of the pair to exceed a First. Instiny distinguished the powers of 2 data add to a, the exists would add the corresponding multiples of b'to get the awar. For example, to multiply (2 by 3) the series would set down the following him:

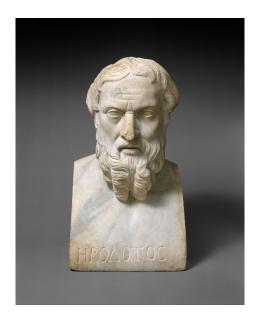
At this point, he would notice that the next doubling would produce 16 in the first column, which is larger than 13. He would then check off those multipliers that added to 13, namely 1, 4, and 8, and add the corresponding numbers in the other column. The result would be written as: Totals 13 156.

Abdress, there is no record of how the scribed did doubling. He snows are simply written down. Polarys he scribe had intermed an extended visit use like in fast, there is some evidence that doubling was a standard method of computation in seas of Africa to the south of Egypt, us in likely that the Egyptian scribes beared from their outlener colleagues. In addition, the scribes were roundow aware that every positive integer could be uniquely expressed as the sum of powers of 2. The fast provides the plantication for the procedure. How won it discovered? Our best guest is that it was discovered by executives the condition of the condition and their conditions are similar to the condition of the condition are conditions.

Because devision is the inverse of multiplication, a problem such as 156×12 we start be stand as "multiply 12×0 as to get 156". The scribe would then write down the stand has been been such as the stand as the stand in the stand as the stand as the stand as the standard performant that sum to 156×166 is this case, 12.4×106 MeV. Then the sum of the corresponding numbers on the left, namely 1.4×106 MeV and 36×106 MeV. Then the sum of the corresponding numbers on the left, namely 1.4×106 MeV and 36×106 MeV a

The last of fractions that the Egyptians used were unit fractions, or "parti," (fractions with amorance 1), with the night exception of 2/2, pulsaps, because there fractions were the most "anternal." The fraction 1/4, (the oth part) is represented in historylytics by the symbol for the integer of with the symbol – shows the father integer is done in section 1. Thus 1/1's denoted in historylytics by #\frac{1}{2}\$ and in the hierarch by \$\frac{1}{2}\$. The single exception, 2.2's, that a practice number is "historylytics and in the hierarch by \$\frac{1}{2}\$. The single exception, 2.2's, that a practice number is "historylytics and in his internal." The forearcy whole is indistinct of the reciprocal of 1/2, \(1 \) for the remainder of this text, however, the notation if "will be such to recrease 1/1 and \(\frac{1}{2} \) or the result of the reciprocal of 1/2.

Herodotus (5th century BC)



Herodotus on Egyptian geometry

It was this king, moreover, who divided the land into lots and gave everyone a square piece of equal size, from the produce of which he exacted an annual tax. Any man whose holding was damaged by the encroachment of the river would go and declare his loss before the king, who would send inspectors to measure the extent of the loss, in order that he might pay in future a fair proportion of the tax at which his property had been assessed. I think this was the way in which geometry was invented, and passed afterwards into Greece . . .

Mediaeval Islamic Egypt



Montucla on ancient Egyptian geometry

HISTOIRE

DES

MATHEMATIQUES,

DANS laquelle on rend compte de leurs progrès depuis leur origine jusqu'à nos jours; où l'on expose le tableau & le développement des principales découvertes, les contestations qu'elles ont fait naître, & les principaux traits de la vie des Mathématiciens les plus célebres.

Par M. MONTUCLA, de l'Académie Royale des Sciences & Belles-Leures de Pruste.

Multi pertransibunt & augebitur scientia. Bâcon-

TOME PREMIER.



A PARIS,

Chez CH. ANT. JOMBERT, Imprimeur-Libraire du Roi pour l'Artillerie & le Génie, rue Dauphine, à l'Image Notre-Dame.

M. D.C.C. L.VIII.

Avec Approbation & Privilege du Ro

DES MATHÉMATIQUES, Part. I. Liv. III. 11
affigner à chacun une portion de terre égale à celle qu'il polfidoit avant l'inondation. Telle fut, divon, l'origine de l'arpentage, première è bauche de la Gómetrie, à laquelle néanmonselle
a donne le nom: car Géometrie, fignifie en Grec, mejure de la
wer, ou dis terrain. Je remarque en paffan que c'êt affice.

a uonne le tioni: car Octonierre, ingine et assert, ou da steraina. Je remarque en passant que celt affez gratuitement qu'on suppose que le Nil confondoit ainsi les limites des possessions; in étote pas bien difficile de lui en oppo-fer d'affez itables ou d'affez profondes pour fubsister maigré l'inondation. On ne se survoir se persuave que l'Egype r'it chaque année ravagée par les eaux : cela s'accorderoit mal avec l'idée d'un pays délicieux, comme celle oue nous en

donne l'antiquité.

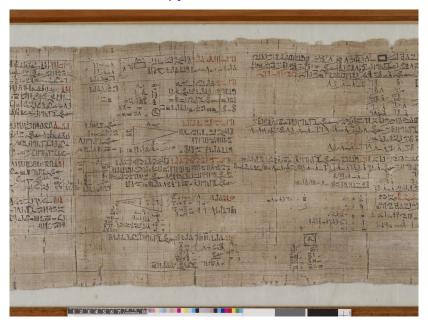
Ouclques Ecrivains, parmi lesquels est Hérodote, fixent la naissance de la Géometrie au tems où Sesostris (k) coupa l'Egypte par des canaux nombreux , & en fit une forte de répartition générale entre ses habitans. M. Newton (1) en adoptant le sentiment d'Hérodote, dit que ce partage sut fait par le conseil de Thot , le Ministre de Sesostris , qui est suivant lui Ofiris. Cette conjecture fur l'emploi & la nature de ce perfonnage célébre, n'est pas destituée d'autorités anciennes, & s'accorde parfaitement avec l'opinion dont on a parlé ailleurs, que Theut étoit l'inventeur des nombres, du calcul & de la Géometrie. En effet, on peut dire que le partage projetté par Sesostris exigeant des connoissances Géometriques, son Ministre en jetta à cette occasion les fondemens. Ceci s'accorde encore avec le sentiment qui attribue ces inventions à Hermes, autrement le fameux Mercure Trismegiste; car tous ces hommes font probablement les mêmes. Un Ecrivain (m) raconte que ce Mercure grava les principes de la Géometrie sur des colonnes qui furent dépofées dans de vaftes fouterrains, & le fabuleux Jamblique (n) dit que Pythagore profita beaucoup de la vûc de ces monumens. Un Auteur enfin cité par Diogene Laerce , (o) dit que Maris , apparemment ce Prince qui fit creuser le fameux lac de ce nom, pour servir de décharge au Nil, avoit inventé les principes de la Géometrie. On voit facilement le motif de sa conjecture.

(h) Herod. l. 11.
(l) Cirue. ed aen. 964.
(m) Amunian. Marcell. revum [cgl. l. (o) In Pythagov, c. 12.
(m) Amunian. Marcell. revum [cgl. l. (o) In Pythagov.

Augustus De Morgan (1838)

There is a *stock history* of the rise of geometry ... that the Egyptians, having their landmarks yearly destroyed by the rise of the Nile, were obliged to invent an art of land-surveying in order to preserve the memory of the bounds of property; out of which art geometry arose. ... There is no proof whatever ...

Rhind Mathematical Papyrus



Reconstructing ancient Egyptian mathematics

- Is it valid to use modern mathematical ideas to reconstruct ancient mathematics?
- What do you do if the mathematical and linguistic evidence point in different directions?
- What story were people trying to tell?
- Where does ancient Egyptian mathematics sit within wider stories?

HT reading course: content

Newton's Universal arithmetick

The reading course will consist of three parts:

- 1. the detailed reading and analysis of Newton's methods;
- 2. the examination of commentaries on Newton's work by British authors Colin Maclaurin and Nicholas Saunderson;
- the investigation of how Newton's ideas were taken up by continental writers, in particular Leonhard Euler and Joseph-Louis Lagrange.

As during the lecture course, the emphasis will be on the use of original sources. You will therefore read selections from the works of the above authors — plus other relevant materials that may arise.

HT reading course: arrangements

Seminars: weekly classes of an hour and a half each (Note that these will timetabled with the lectures as two sessions per week, provisionally on Friday mornings, but you only need to attend one of these — sign up as you would for intercollegiate classes.)

Essays: up to 2,000 words to be submitted in advance for discussion in the seminars in weeks 3, 5 and 7 (Further details will appear online during the vacation.)

Assessment: extended essay (3,000 words), details of which will be announced on Monday of week 7. To be submitted by 12 noon on Monday of week 10.

HT reading course: vacation work

Vacation reading for discussion in first-week seminar is now given on the course webpage.

BSHM

The British Society for the History of Mathematics:

www.bshm.ac.uk

BSHM undergraduate essay prize

http://www.bshm.ac.uk/undergraduate-essay-prize

See you next term...

