Computational Mathematics

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Plan for today

- General information
- Example of `publish`
- The 3 projects

General information

See the Student Guide: hardcopies at reception; online at course website. Project templates are also on the course website.

Weeks 1–2: continuation of last term, with demonstrator sessions scheduled according to college.

Weeks 3–8: "drop-in" demonstrator sessions, especially before deadlines:

- Mondays 3–4pm (Weeks 3–8)
- Thursdays 3–4pm (Weeks 3–8)
- Wednesday 3–5pm (Week 5)
- Friday 3–5pm (Weeks 5 & 8)

General information

Project 1: noon Monday week 6
Project 2: noon Monday week 9

Submit before these deadlines, online via Inspera.

These projects are not very big. Around 6 pages should suffice.

Projects marked for mathematics, computing, and clarity of presentation.

You must prepare your projects alone. Oxford's plagiarism policy applies.

Use MATLAB publish to generate a pdf.

General information

Projects will count towards prelims and marks will be available along with those for prelims exams.

Markers are looking for:

- clear and well-written code which runs;
- code which answers all the questions in the project;
- a submission with both an .m file and a pdf (generated by `publish`);
- plots with legends, axis labels, big enough fonts and clearly distinguished curves (yellow is a really bad colour choice);
- brief explanations of your results.

Publish example

Let's solve a matrix problem of the form $Ax = b$ where $A \in \mathbb{R}^{n \times n}$ and $b \in \mathbb{R}^n$ are a random matrix and vector.

We will investigate how long it takes to solve the problem for different values of n.

The projects

- Project A: Infectious Disease Modelling
- Project B: Quadrature
- Project C: Continued Fractions

Project A: Infectious Disease Modelling

$$\begin{aligned}
\frac{\mathrm{d}S}{\mathrm{d}t} &= -rS(t)I(t) \\
\frac{\mathrm{d}I}{\mathrm{d}t} &= rS(t)I(t) - aI(t) \\
\frac{\mathrm{d}R}{\mathrm{d}t} &= aI(t)
\end{aligned}$$

No exact solution, but can solve using the explicit Euler scheme.

Note $R_0 = S(0)r/a$ is the *basic reproduction rate* and determines whether an epidemic occurs or the disease dies out.

Project A: Infectious Disease Modelling

SIR models were pioneered by Kermack and McKendrick (1927–1933).

Learn more in ASO Mathematical Modelling in Biology and A7 Numerical Analysis courses.

If you want to learn more now, take a look at J. D. Murray: Mathematical Biology I: An Introduction (Springer).

Project B: Quadrature

Look at integrals of the form

$$I(f) \;=\; \int_{-\infty}^{\infty} \mathrm{e}^{-x^2} f(x) \mathrm{d}x$$

Mostly these can't be evaluated exactly, so use approximations called *quadrature rules* of the form

$$I(f) \;\approx\; I_n(f) \;=\; \sum_{j=1}^{n} w_j f(x_j)$$

where the $\{x_j\}$ are the real and distinct nodes, and the $\{w_j\}$ are the positive weights.

Project B: Quadrature

Ideas to back to Gauss (1814), and efficient algorithm developed by Golub & Welsch (1969). More recent work by Trefethen (2022).

Learn more in A7 Numerical Analysis course.

Project C: Continued Fractions

Every real number x can be written as a *continued fraction* in the form

$$\begin{aligned} x &= a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \dots}}} \\ &=: [a_0; a_1, a_2, a_3, \dots] \end{aligned}$$

where $a_k \in \mathbb{Z}$. Here a_0 may be negative or zero, but all other coefficients are positive.

Surprisingly they can be used to find integer solutions of *Pell's equation*: $x^2 - Dy^2 = 1$.

Project C: Continued Fractions

Ideas go back to Bombelli (1572), Cataldi (1613), Wallis (1695), Euler (1737), Lagrange (1768).

Find out more in Carl D. Olds, Continued Fractions (American Mathematical Society, 1963).