EXAMPLES in MODEL THEORY

$\mathcal{A}$	$\mathrm{Th}(\mathcal{A})$	κ-cat.	small	QE	term model	prime model	ctbl. $\aleph_0$-sat.	type omitted
$\langle \mathbb{Q}; < \rangle$	DLO	$\aleph_0$, not $\aleph_1$	✓	✓	–	$\langle \mathbb{Q}; < \rangle$	$\langle \mathbb{Q}; < \rangle$	—
$\langle \mathbb{N}; s; 0 \rangle$	Theory of successor	$\aleph_1$, not $\aleph_0$	✓	✓	✓	$\langle \mathbb{N}; s, 0 \rangle$	$\langle \mathbb{N}; s, 0\rangle \cup \bigcup_{\aleph_0} \langle \mathbb{Z}, s \rangle$	$p(x) = \{\exists y: x = s^m(y) \mid m \in \mathbb{N}\}$
$\langle \mathbb{Q}; +; 0 \rangle$	DTFAG ($\neq 0$)	$\aleph_1$, not $\aleph_0$	✓	✓	–	$\langle \mathbb{Q}; +; 0 \rangle$	$\langle \bigoplus_{\aleph_0} \mathbb{Q}; +, 0 \rangle$	$p(x,y)$ $\{mx \neq ny \mid n,m \in \mathbb{Z}^2 \setminus \{0,0\}\}$
$\langle \mathbb{C}; +, \cdot; 0, 1 \rangle$	ACF_0	$\aleph_1$, not $\aleph_0$	✓	✓	–	$\langle \bar{\mathbb{Q}}; +, \cdot; 0, 1 \rangle$	$\overline{\mathbb{Q}(B)}$ with $\lvert B \rvert = \aleph_0$, B alg. ind.	$p(x) = \{g(x) \neq 0 \mid g \in \mathbb{Q}[x] \setminus \{0\}\}$
$\langle \mathbb{R}; +, \cdot; 0, 1; < \rangle$	RCF	NO	NO	✓ (need "<")	–	$\mathbb{R} \cap \bar{\mathbb{Q}}$	—	$\{0 < x < \frac{1}{m} \mid m \in \mathbb{N}\}$
$\langle K^m; +; 0; (\lambda_k)_{k \in K} \rangle$ K finite	K-vector space ($\neq 0$)	$\lvert K \rvert^m$, $\aleph_0 + \aleph_1$	✓	✓	–	K	$K^{\aleph_0}$	$x_1, \dots, x_{m+1}$ lin. ind.
" $\aleph_0 = \lvert K \rvert$ infinite	"	$\aleph_1$, not $\aleph_0$	✓	✓	–	K	$K^{\aleph_0}$	"
finite structure		$\lvert \mathcal{A} \rvert$-cat.	✓	+/-	+/-	$\mathcal{A}$	–	–
infinite set	"Theory of $\doteq$"	all κ	✓	✓	–	✓	✓	–
$\langle \mathbb{N}; +, 0, 1 \rangle$	Presburger Arithmetic	$\aleph_1$, not $\aleph_0$	✓	✓ with $\equiv_n$ for each n	✓	$\langle \mathbb{N}; +, 0, 1 \rangle$	$\langle \mathbb{N}; +, 0, 1 \rangle \cup \bigcup_{\aleph_0} \mathbb{Z}$	$\{\exists y: x = y + m \mid m \in \mathbb{N}\}$
$\langle \mathbb{Z}; +, \cdot; 0, 1 \rangle$	Arithmetic (not nicely axiomatizable by Gödel Incomp. Thm)	NO	NO	NO	✓ (with "–")	$\langle \mathbb{Z}; +, \cdot; 0, 1 \rangle$	NO	$\{x \neq 0 \wedge m \mid x : m \in \mathbb{N}\}$