B6.1 (NSDE1) - Problem Sheet 1

We study the trajectory of cannon shots. The cannon is positioned at the origin $\mathbf{0} \in \mathbb{R}^2$ of the 2D plane $\mathbb{R}^2$, and it shoots cannonballs with initial velocity $\mathbf{v}_0 \in \mathbb{R}^2$. Cannonballs have mass m and are affected by the gravitational force $\mathbf{F}_G = \binom{0}{-mg}$ and the air resistance $\mathbf{F}_R = -\rho\|\mathbf{v}\|\mathbf{v}$, where $\mathbf{v}$ denotes the cannonball's speed and ρ is the air resistance parameter. By Newton's laws, we conclude that the trajectory of a cannonball, that is, its position $\mathbf{x}(t)$, is governed by the second order initial value problem

$$m\mathbf{x}'' = \begin{pmatrix} 0 \\ -mg \end{pmatrix} - \rho\|\mathbf{x}'\|\mathbf{x}' , \quad \mathbf{x}(0) = \mathbf{0} , \quad \mathbf{x}'(0) = \mathbf{v}_0 . \tag{1}$$

(a) Turn (1) into a first order initial value problem of the form

$$\mathbf{y}' = \mathbf{f}(t, \mathbf{y}) , \quad \mathbf{y}(0) = \mathbf{y}_0 .$$

(b) Show that the right-hand side $\mathbf{f}$ is locally Lipschitz continuous (in the variable $\mathbf{y}$).

(c) Let $g = 10$ and $\rho = 0.1$. Write a MATLAB-function that plots the trajectory of a cannonball with mass $m = 20$ and initial velocity $\mathbf{v}_0 = 100\binom{\cos\pi/9}{\sin\pi/9}$ in the time interval $[0, 5]$ using 1000 explicit Euler steps.

(d) Repeat the previous task, but this time using the implicit Euler method.