PNA - Problem Sheet 2

Exercise 1

Interpolation with Splines

- Implement the function `[coNat] = natural_spline_coeffs(x,y,p)` that returns the matrix of coefficients of a *natural* spline of degree p, where `x` is the vector of interpolation nodes, and `y = f(x)`. The matrix `coNat` should be such that row i gives the spline coefficients of interval $[x_{i-1}, x_i]$ in descending powers of x, e.g.

$$\begin{bmatrix} a_i & b_i & c_i & d_i \end{bmatrix} \Leftrightarrow a_i x^3 + b_i x^2 + c_i x + d_i.$$

- Implement the function `[coPer] = periodic_spline_coeffs(x,y,p)` that returns the coefficients of a *periodic* spline of degree p in the same matrix format.

- Sample the function

$$f(x) = \sin(x) + \cos^2(x) \tag{1}$$

over the interval $[0, 4\pi]$. Plot this function along with both its *natural* spline and *periodic* spline interpolants, for different values of p.

Exercise 2

Least Squares Interpolation We consider 1e4 equispaced samplings of the function

$$f(x) = e^{-x/10} \sin(8x) \cos(7x) + \varepsilon\,, \tag{2}$$

where ε is some random noise (use `noise = @(x)  0.2*rand(size(x))-0.1;`)

- Consider a grid $\{t_i = ih\}_{i=0}^{100}$ with $h = 2\pi/100$. We call *space of piecewise-linear functions* the vector space spanned by the basis functions

$$\{b_i(x) \coloneqq \max(1 - |x - c_i|/h, 0)\}_{i=0}^{100}\,.$$

Compute and plot the piecewise-linear least-squares interpolant of (2) over the interval $[0, 2\pi]$ (for plotting, you may be interested in the MATLAB-function `interp1`.

- Compute and plot the p^{th} degree polynomial least-square interpolant of (2) using a monomial basis $\{x^n\}_{0\le n\le p}$, for $p = 1, 11, 21, \ldots, 81$. Do this with your own implementation (using backslash and/or `qr`) and compare your solutions to those of the MATLAB-function `polyfit`.

- **Challenge:** Find the least-square interpolant of the vector field

$$\mathbf{f}(x, y) = [\sin(x)\cos(y), \sin(y)\cos(x)]^{\mathrm{T}} \tag{3}$$

for $(x, y) \in [0, 2\pi]\times[0, 2\pi]$. Use a multivariate monomial basis $\{x^n y^m\}_{0\le n,m\le 3}$ over (21×21) interpolating points, and plot the results using `quiver`.