B6.1 (NSDE1) - Problem Sheet 3

Exercise 1

Let $C, p > 0$ be fixed constants, and let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ satisfy $|f(h)| = \mathcal{O}(h^p)$ and $|g(h)| = \mathcal{O}(h^{p+1})$, respectively. Show that

$$|Cf(h) + g(h)| = \mathcal{O}(h^p)\,.$$

Exercise 2

The Simpson quadrature rule reads

$$\int_a^b f(\tau)\,\mathrm{d}\tau \approx \frac{b-a}{6}\left(f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right)$$

(a) Construct a 3-stage explicit Runge-Kutta method based on Simpson' quadrature, write its Butcher table, and state its order of convergence.

(b) Write the Butcher table of the collocation Runge-Kutta method based on the Simpson quadrature rule and state its order of convergence.

Exercise 3

(a) Derive the formula of the stability function of the explicit Euler method, of the implicit Euler method, and of the implicit midpoint rule.

(b) Show that the implicit midpoint rule is A-stable.

(c) Show that the implicit Euler method is L-stable.

Exercise 4

(a) Prove that the stability function of any explicit s-stage Runge-Kutta method is a polynomial of degree at most s.

(b) Prove that the stability function of any explicit s-stage Runge-Kutta method of order s is exactly $S(z) = \sum_{j=0}^{s} \frac{z^j}{j!}$.