B6.1 (NSDE1) - Problem Sheet 5

Exercise 1

(a) Write the first and second characteristic polynomials of the explicit Euler method, of the implicit Euler method, and of the implicit trapezium rule.

(b) Show that these methods are zero-stable.

(c) Show that the implicit Euler method and implicit trapezium rule are A-stable using the definition of stability domain of multistep methods.

Exercise 2

Show that

$$hD = \left(\Delta - \frac{1}{2}\Delta^2 - \frac{1}{6}\Delta^3 + \dots \right) E\,,$$

and write the formulas of the first and the second characteristic polynomials of the 1-step and 2-step methods associated to this series. Are these methods zero-stable?

Exercise 3

Prove that a linear multi-step method has consistency order p if and only if $\sigma(1) \neq 0$ and

$$\sum_{j=0}^{k} \alpha_j = 0 \quad \text{and} \quad \sum_{j=0}^{k} \alpha_j j^q = q \sum_{j=0}^{k} \beta_j j^{q-1} \quad \text{for} \quad q = 1, \dots, p\,, \tag{1}$$

and that this condition is equivalent to

$$\rho(e^h) - h\sigma(e^h) = \mathcal{O}(h^{p+1})\,. \tag{2}$$

Exercise 4

Let $a, b \in \mathbb{R}$ be some fixed parameters. Show that the multistep methods described by

$$\rho(x) = (x-1)(ax+1-a)\,, \quad \sigma(x) = (x-1)^2 b + (x-1)a + (x+1)/2$$

are of order 2, and show that they are zero-stable if and only if $a \geq 1/2$.