B1 Set Theory: Problem sheet 0

This is an introductory problem sheet, and solutions to this sheet should not be handed in. The only set theory you've encountered so far may have been in the Mods *Introduction to Pure Mathematics* course, so some of this will be revision from a long way back!

1. Prove that $\varnothing \neq \{\varnothing\}$.

2. (i) Prove that if A, B and C are sets, then $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

(ii) Prove that if A, B and C are sets, then $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

(iii) Prove that if X is a set and A and B are subsets of X, then $X \setminus (A \cup B) = (X \setminus A) \cap (X \setminus B)$.

(iv) Prove that if X is a set and A and B are subsets of X, then $X \setminus (A \cap B) = (X \setminus A) \cup (X \setminus B)$.

3. Let X and Y be sets, let A and B be subsets of X and let C and D be subsets of Y, and let f be a function from X to Y. Which of the following statements are always true?

(i) $f(A) \cup f(B) = f(A \cup B)$,

(ii) $f(A) \cap f(B) = f(A \cap B)$,

(iii) $f^{-1}(C) \cup f^{-1}(D) = f^{-1}(C \cup D)$,

(iv) $f^{-1}(C) \cap f^{-1}(D) = f^{-1}(C \cap D)$,

(v) $f(f^{-1}(C)) \subseteq C$,

(vi) $f^{-1}(f(A)) \subseteq A$,

(vii) $f(f^{-1}(C)) \supseteq C$,

(viii) $f^{-1}(f(A)) \supseteq A$.

4. For the statements in the previous question which are not always true, what conditions on f will ensure that they are true? (eg. f being one-to-one or onto).

5. (i) Prove that if A and B are countable sets, then $A \times B$ is countable.

(ii) Prove that $\mathbb{Z}$ and $\mathbb{Q}$ are countable.

(iii) Prove that the set of finite subsets of $\mathbb{N}$ is countable.

(iv) Prove that $\wp\mathbb{N}$ is uncountable.