

B5.6 Nonlinear Systems

0. Introduction

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Synopsis

1. Geometry of linear systems
Basic concepts of stability and linear manifold of solutions. Orbits in phase-space, linear flows, eigenvalues of fixed points.
2. Geometry on nonlinear systems
Notion of flows, invariant sets, asymptotic sets, attractor.
Conservative and Non-Conservative systems.
3. Local analysis
Stable manifold theorem, notion of hyperbolicity, center manifold.
4. Bifurcation and perturbation.
Bifurcation theory: codimension one normal forms (saddle-node, pitchfork, trans-critical, Hopf)
Nonlinear oscillations through multiple scales.
5. Maps
Poincaré sections and first-return maps. Stability and periodic orbits; bifurcations of one-dimensional maps.
6. Chaos
Maps: Logistic map, Bernoulli shift map, symbolic dynamics,

Examinable?

- The synopsis fully defines what is examinable.
- Emails and questions regarding the material will be gladly and enthusiastically answered.
- Any attempt to know what is examinable (or not), what you should or should not learn for the final paper will be completely ignored.
- If any of the material is not given in the lectures, it will be stated explicitly and removed from the examinable material.
- In the last lecture, I will update you on the examinable material, if there is any change (unlikely).
- Emphasis will be on understanding concepts and applying these ideas to specific systems rather than memorising proofs.

Problem sets

- Four problem sets, starting on Week 4.
- Full solutions are provided.
- Use them or not.
- Do as many as you can/want (hopefully all).
- Discussions in classes will be based on your questions, but will also address finer points of the course.
- Previous papers will be provided.

The lectures

- Gapy slides are provided. Gaps will be filled during the lectures.
- No other notes will be provided.
- References to proofs and more examples in books are given at the end of the slides.
- Lectures start promptly at 14:00 and end at 16:00.
- There will be three breaks. Two mini-breaks (2 1/2 minutes) within the hour.
- One break (often with a special interlude) at the hour.
- The presentation at the interlude is for your own entertainment/education. It is not part of the material.
- You are welcome to listen to them or ignore them.

What you (should) know from previous courses.

- Phase plane analysis for two-dimensional continuous systems

$$\dot{x} = f(x, y) \quad (1)$$

$$\dot{y} = g(x, y) \quad (2)$$

- Poincaré-Bendixson theorem (fixed points, periodic orbits, homo/heteroclinic orbits)
- Basic linear algebra (eigenvalues, diagonalisation,...)