CS-12 Math Physiol ~~2013~~ 2013 q 1 (1

1,

$$\varepsilon \dot{v} = -m^3(v)(0.8-n)(v-1) - \gamma_K n^4 (v+v_K) = g(v,n)$$

$$\tau_n \dot{n} = n_\infty(v) - n$$

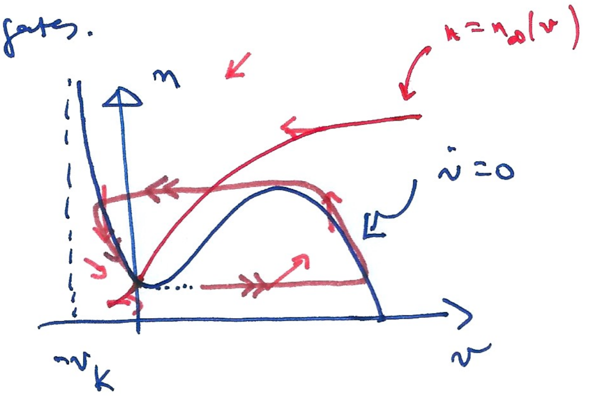
$\varepsilon = 2\times10^{-3}$

$v_K = 0.1$

Equilibrium is $v=0$, $n = n_{eqm}$, unique, stable

(a)(i) n is a gate variable for potassium ion channel transport (each channel has 4 n-gates): n represents the fraction of open n-gates.

(ii)



doesn't ask for proof of this

(iii) with also the n nullcline as above

- large n : $\dot{n}<0$, $\dot{v}<0$

$\Rightarrow$ as indicated by arrows, trajectories cycle round the equilibrium

$\Rightarrow$ $\varepsilon \ll 1$ $\Rightarrow$ as shown in brown a small disturbance causes a large disturbance $\Rightarrow$ excitable

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(b) $\varepsilon v_t = \varepsilon^2 v_{xx} + g(v,n)$

(g as defined top of p 1)

$\tau_n n_t = n_\infty(v) - n$

$v \to 0,\ n \to n_{eq}$ as $x \to \pm\infty$

(i) $y = x - ct$

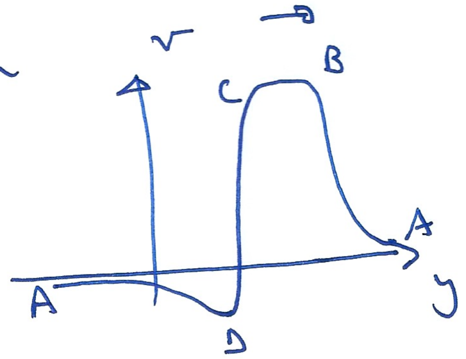
$Y = \frac{y}{\varepsilon}$, $c > 0$

$v(y)$ etc

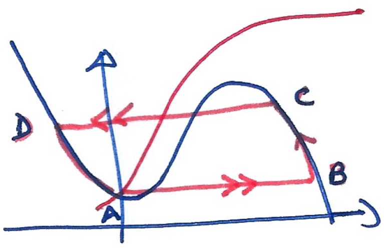
$-\varepsilon c v' = \varepsilon^2 v'' + g$

$-c\tau_n n' = n_\infty - n$

The wave will be as shown



& corresponding trajectory in (v,n) space



The wavefront is a fast phase, $y = \varepsilon Y$

$\Rightarrow \quad -cv' = v'' + g(v,n) \qquad ' = \frac{d}{dY}$

$-c\tau_n n' = \varepsilon(n_\infty - n)$

$\Rightarrow n \simeq n_{eq}$, $\quad v'' + cv' + f(v) = 0$ where

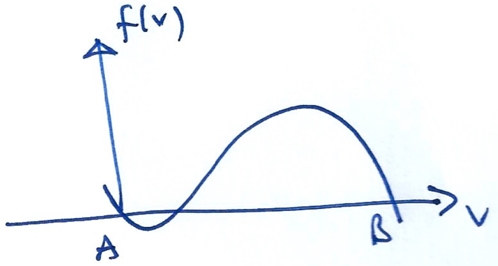
$f(v) = g(v, n_{eq})$

from the figures above

$v \to 0$ as $y \to +\infty$

(ii) Also as $y \to -\infty$ $v \to v_B$ where $f(v_B) = 0$

viz $-m^3(v_B)(0.8 - n_{eq})(v_B - 1) - \gamma_K n_{eq}^4 (v_B + v_K) = f(v_B)$



we note that $-m^3(0)(0.8 - n_{eq}).-1 - \gamma_K n_{eq}^4 v_K = 0$

(since $g(0, n_{eq}) \equiv 0$ & so $f(0) = 0$)

therefore v_B satisfies

$$-m^3(v_B)(0.8 - n_{eq})(v_B - 1) - \left(\frac{v_B}{v_K} + 1\right) m^3(0)(0.8 - n_{eq}) = 0$$

& thus $$v_B - 1 = \left(\frac{v_B}{v_K} + 1\right) \frac{m^3(0)}{m^3(v_B)}$$

and the approximate value of this for small $m^3(0)$ ($m^3(0) \ll v_K m^3(v_B)$)

is $v_B \approx 1$ (the other two values are $v_B = 0$ & $v_B \ll 1$)