C5.12 Math Physiol 2013 q2

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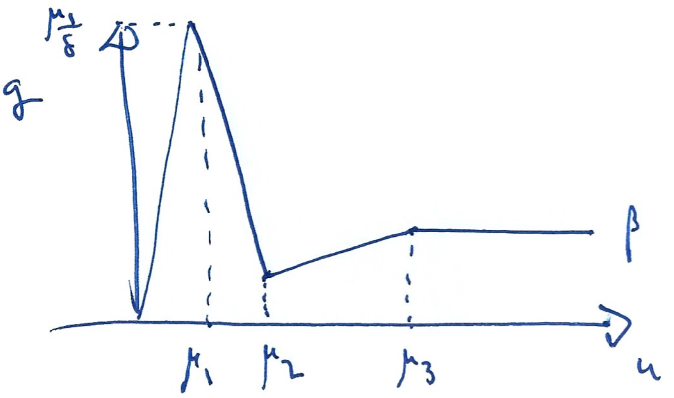
$$\dot{u} + \dot{v} = \mu - u$$

$$\varepsilon \dot{v} = -v + g(u)$$



$\mu_1 = \frac{2}{3}$

$0 < \varepsilon \ll \delta \ll 1$

$\beta = 0.1 \gg \delta$

μ_2 & μ_3 so g is cts.

$$g(\mu_2) = \frac{\mu_1}{\delta} - \frac{1}{\delta}(\mu_2 - \mu_1) = \delta$$

$$g(\mu_3) = \delta + \delta(\mu_3 - \mu_2) = \beta$$

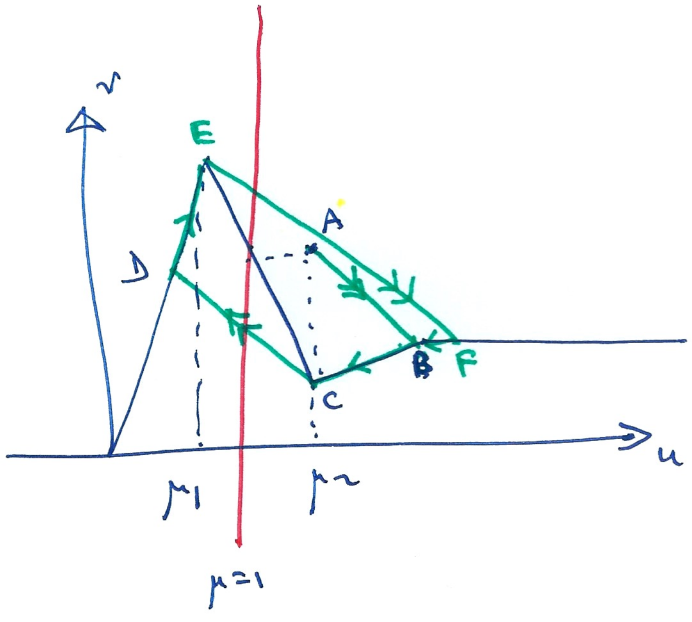
(a) (i) so $\mu_2 - \mu_1 = \mu_1 - \delta^2$ $\quad \mu_2 = \frac{4}{3} - \delta^2$

& $\mu_3 - \mu_2 = \frac{\beta}{\delta} - 1 \Rightarrow \mu_3 = \frac{4}{3} - \delta^2 + \frac{\beta}{\delta} - 1$

(ii) as above

$\mu = 1$ so $\quad \mu_1 < \mu < \mu_2$

(b)



(i) Start at A $(\mu_2, g(1))$

v changes rapidly to get to $v = g(u)$

note at A $v > g(u)$ so $\dot v < 0$ so heads to right + down.

In this rapid change, $u+v \simeq$ constant

AB : rapidly to v nullcline

BC : slowly on v nullcline to $u=\mu_2$ (as $\dot u + \dot v < 0$)

CD : rapidly to v nullcline on $u+v$ constant

DE : slowly to $u = \mu_1$ on v nullcline

EF : rapidly to F on v nullcline along $u+v \simeq$ constant

Then repeat (oscillation) F C D E F....

(ii) On AB for $t \sim \varepsilon$ $\dot u + \dot v \approx 0$ So $u+v \simeq \mu_2 + g(1)$

Well $t_* \sim \varepsilon$. In more detail $t = \varepsilon T$ $(u+v)' \simeq 0$

$\Rightarrow u+v \approx \mu_2 + g(1)$

+ $v' \simeq -v + \delta + S(u-\mu_2)$ at least while $u < \mu_3$

Thus $v' \approx -v + \delta + \delta\,[g(1) - v]$.

Better find where B is.

~~At B~~ At B $\dfrac{v - g(1)}{u - \mu_2} = -1$

$g(1) = \dfrac{2\mu_1 - 1}{\delta} = \dfrac{1}{3\delta}$

So $v = \dfrac{1}{3\delta} - u + \mu_2 = \dfrac{1}{3\delta} + \dfrac{\beta}{\delta} + \dfrac{4}{3} - 1 - \delta^2 - u$

Suppose $v < \beta$ then $\dfrac{\beta + \frac{1}{3}}{\delta} - u + \frac{1}{3} - \delta^2 < \beta$

$\Rightarrow u \approx \dfrac{\beta + \frac{1}{3}}{\delta} > \dfrac{\beta}{\delta} \approx \mu_3$

So B is actually on flat bit.

Well no matter

on AB $\quad v' \approx -v + \frac{1}{3} + O(\delta)$
$(t = \varepsilon T)$

Seems a bit of a vague question since ~~it~~ in this approximation it takes infinite T to get there.

Seems you should just say $t_* \sim \varepsilon$