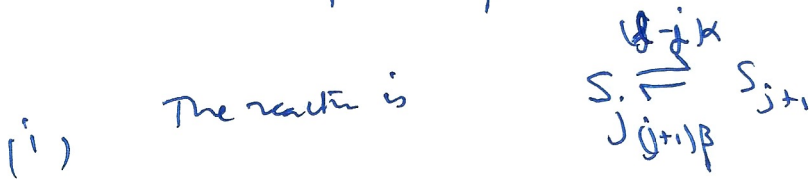
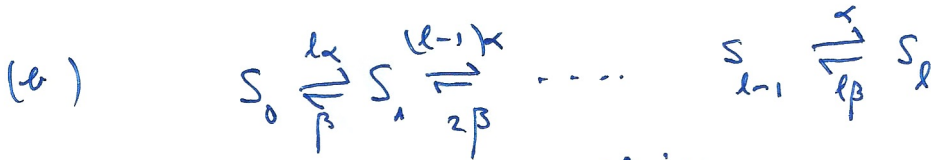




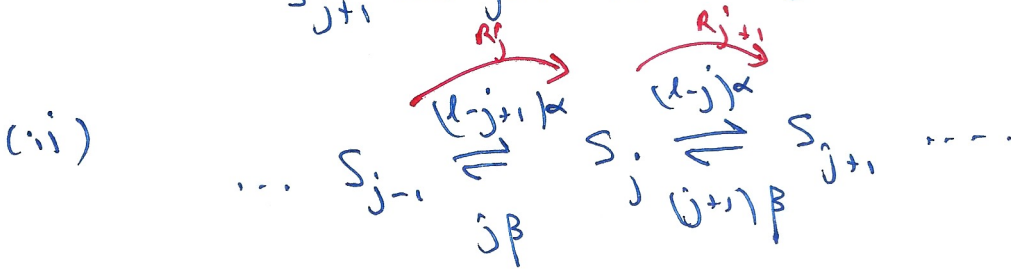
3/ (a) $C \xrightleftharpoons[\beta]{\alpha} O$ ← fraction

$n = \alpha(1-n) + \beta n$
 $= \tau n = n_0 - n \quad \tau = \frac{1}{\alpha + \beta}, n_0 = \frac{\alpha}{\alpha + \beta}$

τ_n relaxation time
 no equilibrium



became S_j has j open ports, $l-j$ closed so $(l-j)$ chances of opening
 S_{j+1} has $j+1$ so $(j+1)$ chances of closing



$x_j =$ fraction of open S_j channels

~~$x_j = \frac{(l-j+1)\alpha}{j\beta}$~~

write $R_j = (l-j+1)\alpha x_{j-1} - j\beta x_j \quad j = 1, \dots, l$

then $x_j = R_j - R_{j+1} \quad j = 1, \dots, l-1$

Also

$$x_0 = -R_1$$

$$x_l = R_l$$

(iii) To show a solution is $x_j = \binom{l}{j} n^j (1-n)^{l-j}$

a cute way is then
~~(using)~~ $\sum_0^l x_j s^j = \sum_0^l \binom{l}{j} (ns)^j (1-n)^{l-j}$
 $= (1-n+ns)^l$

So define $\phi(s,t) = \sum_0^l x_j s^j$

$$\begin{aligned} \text{then } \phi_t &= \sum_0^l \dot{x}_j s^j \\ &= \sum_0^l [R_j - R_{j+1}] s^j \\ &= \sum_0^l R_j s^j - \frac{1}{s} \sum_0^l R_{j+1} s^{j+1} \\ &= \sum_{-1}^l R_j s^j - \frac{1}{s} \sum_0^{l-1} R_{j+1} s^{j+1} \\ &= \sum_{-1}^l R_j s^j - \frac{1}{s} \sum_{-1}^l R_k s^k \\ &= \left(1 - \frac{1}{s}\right) \sum_{-1}^l R_j s^j \end{aligned}$$

provided we define $R_0 = R_{l+1} = 0$

$$\begin{aligned} \text{Also } \sum_{-1}^l R_j s^j &= \sum_{-1}^l (l-j+1)\alpha x_{j-1} s^j - \sum_{-1}^l \beta j x_j s^j \\ &= \sum_0^{l-1} (l-k)\alpha x_k s^{k+1} - \beta s \sum_{-1}^l j x_j s^{j-1} \\ (l-l=0): &= \sum_0^l (l-k)\alpha x_k s^{k+1} - \beta s \sum_0^l j x_j s^{j-1} \quad (0=0) \\ &= \sum_0^l \alpha l s x_k s^k - \sum_0^l \alpha s^2 k x_k s^{k-1} - \beta s \sum_0^l j x_j s^{j-1} \\ &= \alpha l s \phi - \alpha s^2 \phi_{ss} - \beta s \phi_s \end{aligned}$$

Hence

$$\begin{aligned} \phi_f &= \left(1 - \frac{1}{s}\right) [\alpha s \phi - (\alpha s^2 + \beta s) \phi_s] \\ &= (s-1) [\alpha \phi - (\alpha s + \beta) \phi_s] \end{aligned}$$

Let us put $\phi = (1-n+ns)^l$

$$\begin{aligned} \phi_f &= l(1-n+ns)^{l-1} (s-1)n \\ \phi_s &= l(1-n+ns)^{l-1} n \end{aligned}$$

∴ this solves the pde (+ thus the $x_j = e^{y^j}$)

$$\phi_f = l(1-n+ns)^{l-1} (s-1)n$$

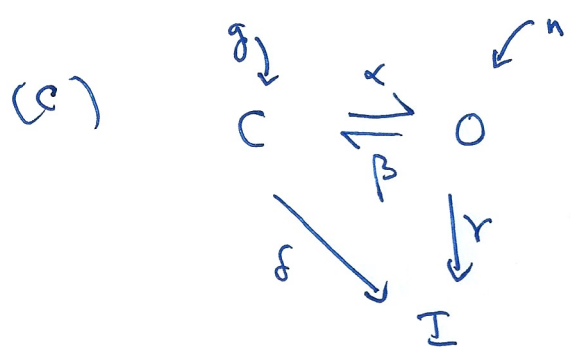
$$= (s-1) [\alpha \phi - (\alpha s + \beta) \phi_s]$$

$$= (s-1) [\alpha l(1-n+ns)^l - (\alpha s + \beta) l(1-n+ns)^{l-1} n]$$

i.e. $\left[\div \text{ by } l(1-n+ns)^{l-1} (s-1) \right]$

$$\begin{aligned} n &= \alpha(1-n+ns) - (\alpha s + \beta)n \\ &= \alpha(1-n) - \beta n \end{aligned}$$

∴



(i) $1 - n - g$

(ii) $\dot{n} = \alpha g - \beta n - \gamma n$

$\dot{g} = -\alpha g + \beta n - \delta g$

(iii) $t \rightarrow \infty \Rightarrow n = 1 \quad g = 0$ all open

$$\begin{pmatrix} \dot{n} \\ \dot{g} \end{pmatrix} = \begin{pmatrix} -(\beta + \gamma) & \alpha \\ \beta & -(\alpha + \delta) \end{pmatrix} \begin{pmatrix} n \\ g \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} n \\ g \end{pmatrix} = \underline{c_1} e^{\lambda_1 t} + \underline{c_2} e^{\lambda_2 t}$$

$$(\lambda + \beta + \gamma)(\lambda + \alpha + \delta) - \alpha\beta = 0 \Rightarrow \lambda_1, \lambda_2$$

$$\Rightarrow g = a(e^{\lambda_1 t} - e^{\lambda_2 t}) \quad (\lambda_1 > \lambda_2) \text{ for } g = 0 \text{ at } t = 0$$

~~then (5e1) $n = \frac{1}{\beta} [g + (\alpha + \delta)g]$~~

then (5e1) $n = \frac{1}{\beta} [g + (\alpha + \delta)g]$

for $t = 0 \quad n = 1 = \frac{1}{\beta} [g|_0] = \frac{a}{\beta} (\lambda_1 - \lambda_2)$

$$\Rightarrow a = \frac{\beta}{\lambda_1 - \lambda_2}$$

$(\text{char } \lambda^2 - (\alpha + \beta + \gamma + \delta)\lambda + \alpha\gamma + \beta\delta + \delta\gamma = 0$

$\Rightarrow \lambda_{1,2} = \frac{1}{2} [T \pm (T^2 - 4D)^{1/2}]$

$$\Rightarrow \lambda_1, -\lambda_2 = (T^2 - 4D)^{1/2} = [(\alpha + \beta + \gamma + \delta)^2 - 4(\alpha\gamma + \beta\delta + \gamma\delta)]^{1/2}$$