STRING THEORY II

LECTURE IX

OXFORD UNIVERSITY

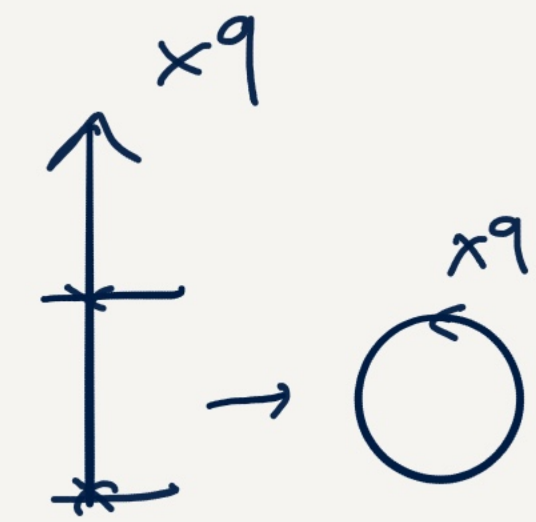
MMATH PHYS TT 2020

GOAL FOR LECTURE 9 & 10: INTRODUCE & STUDY HETEROTIC STRING THEORY.

PREPARATION: TORUS COMPACTIFICATIONS & T-DUALITY & SYMMETRY ENHANCEMENTS.

E.G.: S^1- REDUCTION: AT THE SELFDUAL RADIUS
$U(1) \longrightarrow SU(2)$ SYMMETRY ENCEMENT.

<ins>RECAP</ins>: BOSONIC STRING ON S^1: $X^9 \equiv X^9 + 2\pi R$



<ins>SPACETIME PIC</ins>: KALUZA-KLEIN REDUCTION: (KK)

D dim

$$G_{\mu\nu} = \left(\begin{array}{c|c} \sigma & A_{\mu'} \\ \hline A_{\nu'} & G_{\mu'\nu'} \end{array}\right)$$

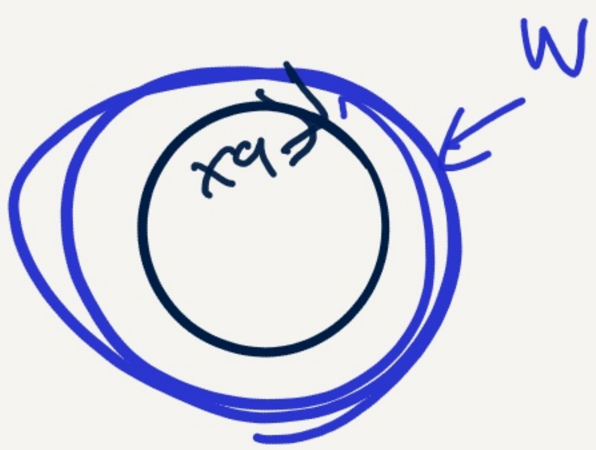
$G_{\mu'\nu'}$ = (D-1) dim.
A U(1) gauge field
σ SCALAR.

GR IN D DIM $\longrightarrow$ GR + YM + SCALAR.

WORLDSHEE PIC: $X^9(\sigma,\tau)$. COMPACTIFIED:

- WINDING MODES: $X^9(\sigma+2\pi,\tau) = X^9(\sigma,\tau) + 2\pi R w$

WINDING NUMBER $w \in \mathbb{Z}$



- MOMENTUM: K^9 HAS TO BE QUANTIZED.

$$e^{2\pi i k^9 R} = 1 \qquad \boxed{K^9 = \frac{n}{R}} \qquad n \in \mathbb{Z}$$

$$P_L^9 = \frac{n}{R} + \frac{wR}{\alpha'} \qquad L_0 = \frac{\alpha'}{4} p_L^2 + N$$

$$P_R^9 = \frac{n}{R} - \frac{wR}{\alpha'} \qquad \tilde{L}_0 = \frac{\alpha'}{4} p_R^2 + \bar{N}$$

$$\Rightarrow \quad \alpha' M^2 = \alpha' \frac{n^2}{R^2} + \frac{w^2 R^2}{\alpha'} + 2\,(N + \bar{N} - 2)$$

$$N + \bar{N} + nw = 0.$$

THE SPACETIME FIELDS $G_{\mu'\nu'}$, σ, $A_{\mu'}$:

$$\alpha_{-1}^{9}\,\tilde{\alpha}_{-1}^{9}|0\rangle \quad \sigma$$

$$\left(\alpha_{-1}^{\mu'}\,\tilde{\alpha}_{-1}^{9} + \alpha_{-1}^{9}\,\tilde{\alpha}_{-1}^{\mu'}\right)|0\rangle \quad A_{\mu'}$$

$$\alpha_{-1}^{\mu'}\,\tilde{\alpha}_{-1}^{\nu'}\,|0\rangle \quad G_{\mu'\nu'}$$

$\exists$ EXTRA STATES THAT ARE NOT JUST THESE KK-STATES

$\Rightarrow$ MOMENTUM / WINDING STATES:

$$\exp(i\,p_L\cdot X_L + i\,p_R\cdot X_R)\,|0\rangle \equiv |p_L, p_R\rangle.$$

$$X = X_L(z) + X_R(\bar{z})$$

EXACT SYMMETRY OF THIS CFT (BOSON ON S^1)

T-DUALITY.

$$\boxed{R \longrightarrow \frac{\alpha'}{R} = R' \qquad n \longleftrightarrow w}$$

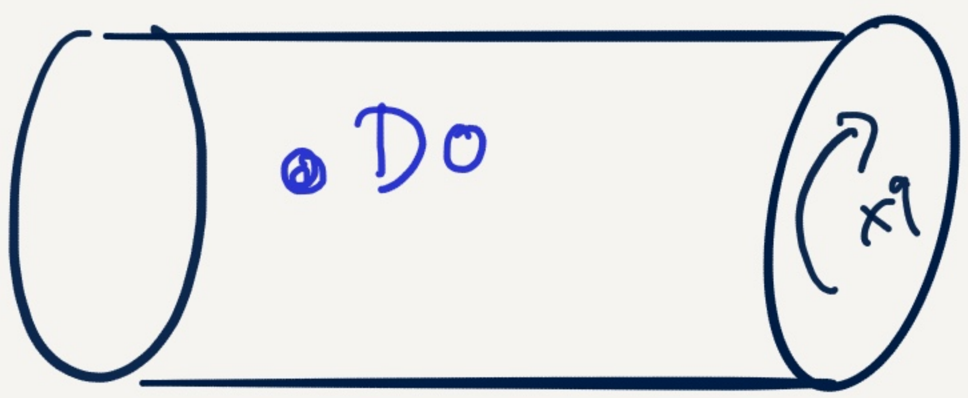
$$X_L + X_R \rightarrow X_L - X_R \qquad \begin{array}{l}\alpha \rightarrow \alpha \\ \tilde{\alpha} \rightarrow -\tilde{\alpha}\end{array}$$

D-BRANES:

T-DUALITY ACTION ON D-BRANES DEPENDS ON WHETHER WE DUALIZE ALONG THE WORLD-VOLUME OR TRANSVERSE.

$$|D0\rangle\rangle = \exp\left(\frac{1}{n} \sum \alpha^9_{-n} \tilde{\alpha}^9_{-n}\right) |0\rangle$$

$$|0\rangle = \sum_{n \in \mathbb{Z}} e^{i n/R} \underbrace{|0,n\rangle}_{\text{momentum } n \text{ state } (w=0).}$$



$$\alpha \to \alpha$$
$$\tilde{\alpha} \to -\tilde{\alpha}$$

$$\exp\left(-\frac{1}{n} \sum \alpha^9_{-n} \tilde{\alpha}^9_{-n}\right) |0\rangle'$$

$$= |D1\rangle\rangle.$$

$$|0\rangle' = \sum_{w \in \mathbb{Z}} e^{\frac{i w R}{\alpha'}} |w,0\rangle$$

winding state.

$$Dp \xrightarrow[\text{ALONG WORLD-VOLUME (WV)}]{\text{T-DUALITY}} D(p-1)$$

$$Dp \xrightarrow[\text{TRANSVERSE TO WV}]{\text{T-Duality}} D(p+1).$$

WE WANT TO STUDY RNS-STRING ON S^1 & T-DUALITY:

$$\begin{aligned} X_L &\to X_L \\ X_R &\to -X_R \end{aligned} \xrightarrow{\text{SUSY}} \begin{aligned} \psi &\to \psi \\ \bar{\psi} &\to -\bar{\psi} \end{aligned} \qquad \text{T-DUALITY.}$$

$\Rightarrow$ UNDER T-DUALITY THE RIGHT-MOVING R-SECTOR CHANGES CHIRALITY OF THE GROUND STATE!

$X^9, \psi^9, \tilde{\psi}^9$ COMPACTIFIED ON S^1.

AS ALWAYS IN R-SECTOR: GROUNDSTATE:

$$\circledast \left[\begin{array}{l} \Gamma^i_\pm = \frac{1}{2}\left(\tilde{b}^i_0 \pm i\, b^{i+4}_0\right) \qquad i = 2 \ldots 5 \\ \Gamma^5_\pm = \frac{1}{2}\left(\tilde{b}^5_0 \pm i\, \tilde{b}^9_0\right). \end{array} \right\}$$

RAISING, LOWERING OPS. ON THE RIGHT MOVERS.

$\Rightarrow$ T-DUALITY: $\tilde{b}^9_0 \rightarrow -\tilde{b}^9_0$

$$\Gamma^5_+ \rightarrow \Gamma^5_-$$

$\Rightarrow$ EXCHANGES R-SECTOR RIGHT MOVING SPINOR REP

$$|a\rangle \leftrightarrow |\dot{a}\rangle.$$

$\Rightarrow$ $\mathrm{II\,A} \leftrightarrow \mathrm{II\,B}.$

$$\boxed{\mathrm{II\,A} \text{ ON } S^1_R \xleftrightarrow{\text{T-DUAL}} \mathrm{II\,B} \text{ ON } S^1_{\frac{\alpha'}{R} = R'}}$$

NOTE: CONSITENT w/ D branes:

$$D(2p) \longrightarrow \left\{ \begin{array}{l} D(2p+1) \\ D(2p-1) \end{array} \right.$$

MORE GENERALLY: T-DUALITY ON A $(S^1)^D = T^D$ TORUS

$$(10-D)d \ \text{IIA} \ \xrightarrow{T^D} \ \begin{cases} (10-D)d \ \text{IIA} & D \text{ EVEN} \\ (10-D)d \ \text{IIB} & D \text{ ODD.} \end{cases}$$

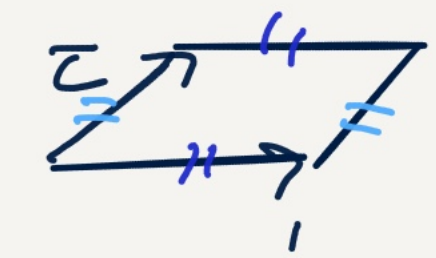
T-DUALITY IS A SYMMETRY OF CFT OF THE BOSONS?:

PARTITION FUNCTION REMAINS INVARIANT:

Z_{T^2} FOR $X^9(z,\bar{z})$ on S^1_R:

$q = e^{2\pi i \tau}$

T^2



$$Z_{T^2} = \frac{1}{(q\bar{q})^{\frac{1}{24}}} \, \mathrm{Tr}\, q^{L_0} \bar{q}^{\bar{L}_0}$$

$$= \frac{1}{|\eta(q)|^2} \sum_{n,w \in \mathbb{Z}} q^{\frac{\alpha' p_L^2}{4}} \, \bar{q}^{\frac{\alpha' p_R^2}{4}}$$

SEE PARTITION FU DISCUSSION EARLIER: LECTURE 4?

α_{-n} & $\tilde{\alpha}_{-n}$ OSCILLATOR CONTRIBUTIONS.

$$Z_{T^2} = \frac{1}{|\eta(q)|^2} \sum_{n,w\in\mathbb{Z}} \exp\left(-\pi\tau_2\left(\frac{\alpha' n^2}{R^2} + \frac{w^2R^2}{\alpha'} + \tau_1 2\pi i n w\right)\right).$$

$\tau = \tau_1 + i\tau_2$

MANIFESTLY T-DUALITY INVARIANT.

CHECK THAT IT IS ALSO INV. UNDER MODULAR TRANSFORMATIONS (PSZ).

$$\left.\begin{aligned} T:&\quad \tau \to \tau + 1 \\ S:&\quad \tau \to -1/\tau \end{aligned}\right\}$$

GENERATE $SL(2,\mathbb{Z})$ MODULAR GROUP $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ INTEGER

$$\tau \longrightarrow \frac{a\tau+b}{c\tau+d}$$

POISSON RESUMMATION:

$$\sum_{n\in\mathbb{Z}} \delta(x-n) = \sum_{k\in\mathbb{Z}} e^{2\pi i k x}$$

$\int dx\, e^{-\pi a x^2 + bx}$:

$$\sum_{n\in\mathbb{Z}} e^{-\pi a n^2 + bn} = \frac{1}{\sqrt{a}} \sum_{m\in\mathbb{Z}} e^{-\frac{\pi\left(m - b/2\pi i\right)^2}{a}}.$$

$$Z_{S^1_R} = \underbrace{\frac{1}{|\eta(q)|^2}\,\frac{1}{\sqrt{4\pi^2\alpha'\tau_2}}}\; 2\pi R \cdot \sum_{n,w\in\mathbb{Z}} e^{-\frac{\pi R^2\,|n-w\tau|^2}{\alpha'\tau_2}}$$

MODULAR INV.
BY ITSELF

$T: \quad \tau \to \tau+1: \qquad n \to n+w \qquad$ RELABELING.

$S: \quad \tau \to -\frac{1}{\tau}: \qquad n \to -w \qquad w \to n.$

NEXT LECTURE: HIGHER DIM TORUS

$$T^N = \cancel{(S^1)^N} = \mathbb{R}^N/\Lambda$$

$$\Lambda \cong \mathbb{Z}^N$$

$$X^\mu \simeq X^\mu + \lambda^\mu \qquad \lambda^\mu \in \Lambda.$$

$N=1: \quad \lambda^\mu \hat{=} 2\pi R w.$

ASSOCIATED TO Λ: DEFINE MOMENTUM & WINDING LATTICE (NARAIN LATTICE).

$$\underline{\ell} = (\underbrace{\underline{\ell}_L}_{N\text{-vector}}, \underbrace{\underline{\ell}_R}_{N\text{-vector}}) \in \Gamma \cong \mathbb{Z}^{2N}$$

$$\left.\begin{aligned} \ell_L^i &= \sqrt{\frac{\alpha'}{2}}\, k_L^i = \sqrt{\frac{\alpha'}{2}}\left(\frac{n^i}{R_i} + \frac{w^i R^i}{\alpha'}\right) \\ \ell_R^i &= \sqrt{\frac{\alpha'}{2}}\, k_R^i = \sqrt{\frac{\alpha'}{2}}\left(\frac{n^i}{R_i} - \frac{w^i R^i}{\alpha'}\right) \end{aligned}\right\} \text{N COPIES OF } S^1 \text{ MOMENTA.}$$

QUESTION: WHAT ARE THE CONDITIONS ON Λ SUCH THAT THE RESULTING COMPACTIFIED STRING IS WELL-DEFINED?

- $\exp(ik_L X_L + ik_R X_R)\,|\ell\rangle$ DO NOT HAVE BRANCHCUTS.
- Z_{T^N} IS MODULAR INVARIANT.