

Using the θ -method to solve ODEs: List of Corrections

MMSC Cohort 2023–24

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The numbers in the list below correspond to the handwritten numbers in circles on the marked up scan of the “badly written document”. I agree that some of the text does not flow as well as it might. This is a consequence of trying to write a rather brief document with a certain set of mistakes in. Here is the list of errors we found in class.

1. No need for an apostrophe in ODE’s.
2. It’s is a contraction of it is. Here the meaning was belonging to it which is written its.
3. Typo — should have been “achieved”
4. The derivative in (1) should have been written

$$\frac{du}{dt} = f(t, u)$$

with a Roman font for the d in the numerator and denominator. Similarly a Roman font should be used for the d in an integral, e.g.

$$I = \int_0^1 x^2 dx$$

The Roman font can be achieved in L^AT_EX using `{\rm d}`.

5. $f(t, u)$ should have been in maths font (e.g. using `$ $`) and written as $f(t, u)$. Maths in text should always have a maths font.
6. Again a maths font was needed but the point here was that the order of the arguments in a function should be consistent.
7. The correct spelling is “spatial”.
8. The domain should have been written as $(-1, 1) \times (0, T]$. Use `\times` in L^AT_EX to get the \times .
9. As well as making sure you write inline maths in a maths font, you need to remember to write the text in an equation in a Roman font. Here “for” needed a Roman font, e.g.

$$u(x, 0) = u_0(x) \quad \text{for } -1 < x < 1,$$

(use `{\rm for}`). It might also have been nicer to use an array to ensure all the occurrences of the word for are lined up.

10. Avoid abbreviations. It is better to write do not, rather than don't, in formal written reports.

11. The symbol θ should be bold in the section title. To achieve this use

```
\section{The  $\{\boldsymbol{\theta}\}$ -method}.
```

This also has the effect of making the θ boldface in the table of contents if you have one. This might not be what you want. It depends on what level of section title you have. In an article section titles are bold, anything lower is not bold. You can get around this using

```
\section[The  $\theta$ -method]{The  $\{\boldsymbol{\theta}\}$ -method}
```

The extra argument in the square brackets appears in the table of contents, the original argument is the usual section title. A similar approach can be taken with figure captions so that they can be shortened in a list of figures.

12. A hyphen is missing in θ -method.

13. There should be a comma after “for $n = 0, \dots, N$ ”.

14. The brackets are missing from around the equation number (remember to use `\eqref`).

15. We should have U_{n+1} rather than $U_n + 1$. Use `U_{n+1}` rather than `U_n+1`.

16. No need for a capital letter, we are not starting a new sentence.

17. “where θ is between 0 and 1” is ambiguous (does it include the end points or not?). It might be better to say “where $\theta \in [0, 1]$ ”.

18. As a rule sentences should not start with a symbol or number. Here, writing “Three” would solve the problem, otherwise sentences should be reordered.

19. Leave a space between a word and a (.

20. Open quotation marks correctly. \LaTeX has a nasty habit of making them all look like closing quotation marks. Use ‘ ‘ and ’ ’.

21. First two items in list need a semi-colon (;) at the end.

22. Crank-Nicolson usually has a hyphen (remember to be consistent with this).

23. Follow the order section, subsection, subsubsection. Here the subsection was missing giving the weird subsubsection number 2.0.1. This should have been a subsection.

24. There is no need for indentation after (2). This has occurred because of a blank line after the equation in \LaTeX . Avoid these!

25. The inline equation should not be split over multiple lines if possible. There's an argument that this should have just been a standard equation.

26. The term $1/2\Delta t$ is unnecessarily confusing. It is not clear if we mean $1/(2\Delta t)$ or $\Delta t/2$ (of course it is the latter). Using brackets if there are multiple terms in the denominator, or reordering terms helps remove ambiguity.

27. It is inconsistent to use $\Delta t/2$ in the first line of the equation and $\frac{\Delta t}{2}$ in the second line.

28. Punctuate the equations. They end the sentence so put a full stop at the end.

29. Probably no need for a new paragraph or a colon before the equation.

30. No need for a new paragraph.

31. Be consistent about whether you refer to (3) or Equation (3) rather than using both very close together.

32. Having lots of brackets all the same size makes it really hard to see which brackets pair up. Use `\left(` and `\right)` to get variable sized brackets. A neater version of the equation is

$$T_n = \frac{1}{\Delta t} \left\{ \left(u(t_{n+1/2}) + \frac{\Delta t}{2} u'(t_{n+1/2}) + \frac{1}{2} \left(\frac{\Delta t}{2} \right)^2 u''(t_{n+1/2}) \right) - \left(u(t_{n+1/2}) - \frac{\Delta t}{2} u'(t_{n+1/2}) + \frac{1}{2} \left(\frac{\Delta t}{2} \right)^2 u''(t_{n+1/2}) \right) \right\} - \theta \left(u'(t_{n+1/2}) + \frac{\Delta t}{2} u''(t_{n+1/2}) \right) - (1 - \theta) \left(u'(t_{n+1/2}) - \frac{\Delta t}{2} u''(t_{n+1/2}) \right) + \mathcal{O}(\Delta t^2).$$

33. The last line of the equation is too long and overwrites the equation number. Also the font for $\mathcal{O}(\Delta t)$ is wrong. It should be $\mathcal{O}(\Delta t)$ (use `\mathcal{O}`).
34. Insert comma after “shown”.
35. Typo — too many “the”s. (Very easy to overlook!)
36. Ideally add some commas

$$T_n = \begin{cases} \mathcal{O}(\Delta t), & \text{for } \theta \neq 1/2, \\ \mathcal{O}(\Delta t^2), & \text{for } \theta = 1/2, \end{cases}$$

37. Perhaps also reorder the equation so the $\theta = 1/2$ case comes first to match the next sentence.
38. *theta* should be θ (remember the backslash in `\theta`).
39. (??) should have been sorted out — the cross referencing here was to an equation label which did not exist.
40. The first two equations should have been lined up so that the equals signs were one below the other. Avoid using the \implies sign. Generally there should be words to explain what is going on. A suitable correction here would be

$$U_{n+1} = U_n + \Delta t (\theta f(t_{n+1}, U_{n+1}) + (1 - \theta) f(t_n, U_n)), \quad (1)$$

$$u_{n+1} = u_n + \Delta t (\theta f(t_{n+1}, u_{n+1}) + (1 - \theta) f(t_n, u_n)) + \Delta t T_n. \quad (2)$$

Subtracting (1) from (2), taking the modulus and applying the triangle inequality gives

$$|u_{n+1} - U_{n+1}| \leq |u_n - U_n| + \theta \Delta t |f(t_{n+1}, u_{n+1}) - f(t_{n+1}, U_{n+1})| + (1 - \theta) \Delta t |f(t_n, u_n) - f(t_n, U_n)| + \Delta t |T_n|.$$

41. The correct hyphenation is “right-hand side”.
42. Maybe insert a space after \forall . Also Ω was never defined.
43. Use `\ll` in `\LaTeX` to get better spacing. The spacing of \ll (using `\ll`) is better than \ll (using `<<`).
44. Use `\max` to get the correct font and a better subscript, as in

$$T_{\max} = \max_{0 \leq n \leq N} |T_n|$$

(Similarly remember `\sin`, `\exp`, `\log` etc.)

45. Be consistent about whether to write $n = 0, 1, \dots, N$ or $n = 0, \dots, N$.
46. There is extra space before the equation caused by a blank line in the .tex file.
47. The notation Δt has changed to dt . Remember to be consistent about notation. (This change continues in the next equation along with the $U_n + 1$ mistake from earlier.)
48. Typo — implicit is spelt wrongly.
49. The $:=$ symbol should be \coloneqq (use `\coloneqq` from the mathtools package).
50. Newton Rhapsody should be Newton–Raphson (with dash and correct spelling of Raphson).
51. Since we already used “summarise” with the British spelling “summarized” should also have an British spelling. Try to stick with either British English spelling or American English spelling rather than switching between both.
52. Make sure your Matlab code does not have lines which are too long.
53. Be consistent about naming — the names Newton-Raphson method and Newton’s method appear. Pick one and stick to it!
54. Use `\log`. While $\log \log(4 + u^2)$ is much easier to read, perhaps $\log(\log(4 + u^2))$ is even clearer.
55. There is no need for brackets around figure numbers.
56. Figure 1 is far too small. The legend would have been better in the top left or bottom right, i.e. not overlapping the curves. There should also be details of the numerics so that someone could reproduce the results. For example what is Δt ? What is the tolerance for Newton’s method?
57. Never refer to just “the figure” or “the figure below”. Refer to each figure specifically by its number.
58. The second occurrence of implicit Euler should have been Crank–Nicolson.
59. Figure 2 is also too small but the new error here is that it is not discussed in the text. All figures should be discussed in the text so that the reader knows when to look at them and what to learn from them. It could be argued that this solution should have been on Figure 1, or that these figures could be next to each other.
60. The caption for Figure 2 needs to be more descriptive and the quotation marks need sorting out.
61. Using yellow as a colour for a line on a graph is not a good idea. It can be very hard to see. Ideally an accessible colour scheme would be used.
62. Much of the caption for Figure 3 could have been moved to the main body of the text.
63. It is a bit odd that the first time “ordinary differential equation” is written out in full is in the final paragraph.
64. Avoid using `\frac` in text. It is better to write $1/2$.
65. Typo — order is spelt wrongly.
66. The references are inconsistent and incomplete. In particular:
 - Use “and” rather than `&` in the second reference;
 - Make the ordering of names consistent between references (i.e. always initials then surname or vice versa);

- Capitalise all words in the title of the second reference.
- The second reference needs a publisher and date.

Optional extras are ISBN/doi. As well as authors and title, journal articles should always have the name of the journal, the volume (and number if appropriate), pages and year. Webpages should have the URL and date accessed.

As a final comment: be consistent about capitalisation in titles. There are two approaches: capitalise everything (apart from minor words like “a”, “of”, “the”, . . .); or capitalise the first word and proper nouns. Either approach is fine but be consistent throughout a document.